The test covers material in chapter 4 and chapter 5 that we covered in class.

4.1 Continuous RVs.

1. What is a density, give an example
2. How does a density answer a question about probability?

Exercise 1

$$f(x) = \begin{cases} c(1-x^2) & 0 < x < 1 \\ 0 & o/w \end{cases}$$

1. *Find c*
2. *Find $P(1/2 < X)$*
3. *Find $F(x) = P(X < x)$*
4. *Find x so that $F(x) = 1/2$.*

Expectation of X 1. What does the term expectation of X mean?

2. Why might one say the expectation is the center?

Exercise 2 *1. Find $E(X)$*

2. Find $Var(x)$.

The uniform 1. Why is the uniform called *uniform*?

2. Suppose a traffic light stays red for 90 seconds. If you drive up to a red light, explain why one might argue that the time you have to wait is uniform. What parameters?

Exercise 3 *1. Let X be uniform on $[0,1]$. Explain why the variance is* `more` *than the last answer.*

2. If U is uniform on $[10,100]$ find $F(x)$

3. Use the above to find the 25th percentile

Exponential 1. Why is this distribution given its name

2. What is the memoryless property?
3. WHy is the integral for Γ so useful here?

Exercise 4 *1. Let X be exponential with rate 10. Find the mean and median.*

2. *Let X be exponential with rate 100, find $P(X > 100)$*

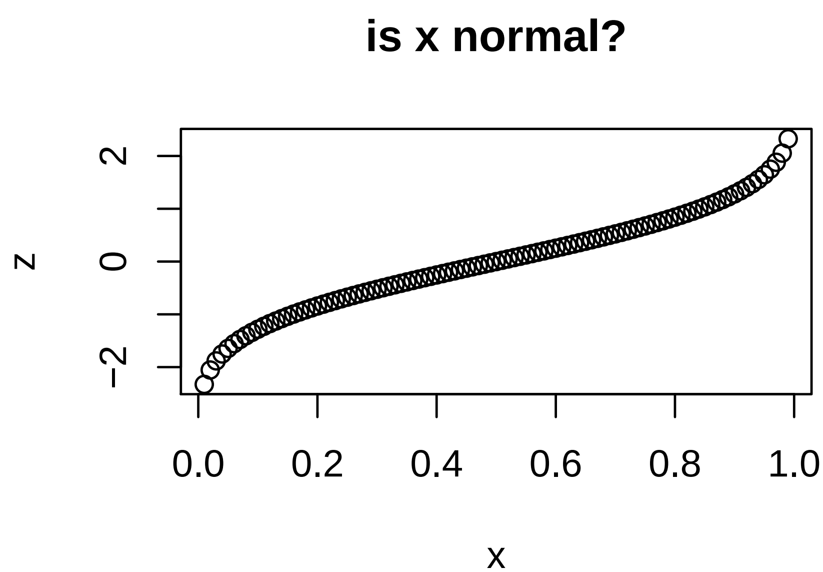
Gamma Exercise 5 *Use the gamma integral to find $E(X^4)$ when X is exponential with rate 1.*

Normal 1. Why is the normal normal?

2. How can you identify σ from the graph of the density?
3. Know the rules of thumb.

Exercise 6 1. *Use table to find $P(-.5 < Z < .75)$.*

2. *Which is greater $P(Z > 1)$ or $P(-1 < Z < 0)$?*
3. *If X is normal with mean 270 and sd 7 find $P(260 < X < 275)$.*
4. *For X above, find the 75th percentile*
5. *Find $E(X^4)$ using the gamma integral and substitution.*
6. *The quantile plot is of some distribution against the normal. Can you tell if the distribution is normal too?*



Joint - discrete 1. Know how to relate a table of numbers to a probability question, like $P(X = x, Y = y)$.

Exercise 7 *1. Let the joint distribution of X (on row) and Y (column) be given by the following:*

```
> m
   Y
X        -1      0      1
  1 0.0667 0.2667 0.0667
  2 0.1333 0.0667 0.1333
  3 0.0667 0.1333 0.0667
```

Find the following:

(a) The marginal distribution of X

(b) The marginal distribution of Y.

(c) The probability $P(X \leq 2, Y \geq 0))$

(d) The conditional probability $P(X \leq 2 \mid Y = 1)$

Joint Continuous 1. The issue here is setting up the integrals!!!

1. Let $f(x) = c(2-(x+y))$ over the unit square: $0 < x, y < 1$. Find c
2. (U,V) is a randomly chosen point over the unit circle $x^2 + y^2 < 1$. Find the joint density:
3. Pick U and V uniform on the interval $[0,1]$. Find the probability that $|U - V| < 1/2$.

Conditional probability 1. There are similar – but different – formulas for discrete and continuous

Exercise 8 *1. Suppose X and Y are uniform on 1,2,3,4. Find the p.d.f of $X+Y$ by computing*

$$f(k) = \sum P(X + j = k | Y = j) P(Y = j)$$

2. Suppose X and Y are uniform on $[0,1]$ – and independent

(a) Find $f(X = x | Y = y)$.

(b) Use that to compute the density of $X+Y$

$$f_{X+Y}(x) = \int_0^1 f(X = x - y | y = y) f_Y(y) dy$$

Independence Independence means multiply! For statistics it is a key example, as then the joint distribution of a sample from the same population will have joint density

$$f(x_1, x_2, ..., x_n) = f(x_1) f(x_2) \cdots f(x_n)$$

which with luck is manageable.

To check independence is different. We need to show for all cases that the joint p.d.f. is a product of the marginal.

Exercise 9 *1. For the following table:*

```
> m
   Y
X        -1      0      1
  1 0.0769 0.1538 0.0769
  2 0.1538 0.0769 0.1538
  3 0.0769 0.1538 0.0769
```

Are X and Y independent?