311 review sheet. These are some problems to work one to help prepare you for the exam. They are not meant to be templates for actual exam questions.

The exam will cover chapters 2 and 3 from the textbook, as much as it appeared in the syllabus.

I will give the following formulas. (If you want more, let me know and we can "negotiate.")

$P(E) = \#E/\#S)$ for equally likely outcomes and finite sample spaces; Definition 2.3; Thm 2.1; $P_r^n = n!/(n-r)!$; $\binom{n}{r} = n!/(r!(n-r)!)$; $P(A|B) = P(AB)/P(B)$; Independence means $P(AB) = P(A)P(B)$; $P(A \cup B) = P(A) + P(B) - P(AB)$; A discrete RV has distribution $p(x) = P(X = x)$ satisfying $p(x) \geq 0$ and $\sum p(x) = 1$; $E(X) = \sum xP(X = x)$; $E(g(X)) = \sum g(x)P(X = x)$; $V(X) = E([X - E(X)]^2) = E(X^2) - E(X)^2$; $P(X > a) \leq P(|X| > a) = P(|X - \mu|/\sigma > |a - \mu|/\sigma) \leq ((a - \mu)/\sigma)^{-2}$;

As well the named distributions:

Bernoulli(p) $P(X = x) = p^x(1-p)1-x$; $E(X) = p$, $V(X) = p(1-p)$

Binomial(n,p) $P(X = k) = \binom{n}{k}p^k(1-p)^{n-k}$; $E(X) = np$; $V(X) = np(1-p)$

Geometric(p) $P(X > k) = (1-p)^k$; $E(X) = 1/p$; $V(X) = (1-p)/p^2$

Negative binomial(r,p) $P(X = k) = \binom{k-1}{r-1}p^r(1-p)^{nk-r}$; $E(X) = r/p$; $V(X) = r(1-p)/p^2$

Poisson(λ) $P(X = k) = \lambda^k/k!e^{-\lambda}$; $E(X) = V(X) = \lambda$.

Hypergeometric(N,k,n,r) $P(X = x) = \binom{k}{x}\binom{N-k}{n-x}/\binom{N}{n}$; $E(X) = nk/N$; $V(X) = nk/N(1 - k/N)(N-n)/(N-1)$

1. A drug manufacturer is testing the effectiveness of a new drug and is worried about the "placebo effect". A group of 100 people with the infection is split into two groups of 50: a treatment group and a placebo group. The treatment group is given the drug and the placebo group is given a sugar pill that looks like the drug. Suppose it is found that 10 people in the treatment group got better and 6 people in the placebo group got better.

   (a) If you pick a person at random, what is the probability that that person was in the treatment group?

   (b) If you pick a person at random, what is the probability that that person got better?

   (c) If you pick a person at random, given that that person got better, what is the probability they were in the treatment group?

   (d) If you pick a person at random, given that that person was inG the treatment group, what is the probability they got better?

2. A survey is done about which computer operating system(s) people use. It is determined that

   - 100% of the people use either Windows, Macintosh or Linux.
   - 95% use Windows.
   - 10% use Macintosh.
   - 25% use Linux.
   - 0% use both Macintosh and Linux.
   - 8% use both Macintosh and Windows.
   - 22% use both Linux and Windows.

   A person is selected at random from this group and asked which operating system s/he uses.

   (a) Draw a Venn diagram illustrating this. Shade in the region corresponding to the event that the person uses exactly one of these operating systems.
   (b) What is the probability that a person uses Windows or Macintosh?
   (c) What is the probability that the person use only Linux?
   (d) What is the probability that the person uses all three operating systems?

3. Suppose the probabilty of A is $P(A) = .6$, and the probablity of B is $P(B) = .3$.

   (a) Suppose you know that A and B are disjoint events. What is $P(A \cup B)$?
   (b) Suppose you know that A and B are independent events. What is $P(A \cup B)$?
   (c) Suppose that all you know about A and B are the two probabilities above (.6 and .3), what is the smallest value that $P(A \cup B)$ can be?

4. A computer programmer uses a random 8 digit number to encode a web-site transaction. If there are 1000 transactions, write down an expression that gives the probability that all 1000 numbers are distinct.

5. A multiple choice test has 5 possible answers for each question only one of which is correct.

   (a) Suppose a student knows 60% of the correct answers. If a student knows an answer, they write the correct answer down otherwise they guess at random from the 5 possible answers. If a question is chosen at random, what is the probability that the student got the answer correct?
   (b) Now suppose another student knows only 50% of the answers, but when guessing always eliminates 3 answers so that s/he guesses between exactly 2 possible answers. Again, if a question is chosen at random from the test, what is the probability that this student got the answer correct?

6. Two teams play a best 3 out of 5 (first team to win 3 games). If team A has a $p \cdot 100\%$ chance of winning each game, what is the probability team A is the first team to win 3 games? (Is it more or less than p if $p > 1/2$?)

7. A driver notices 5 cars at a light in front of them. They figure they will make the light if no more than 2 of the 5 turn left. The driver figures that each car will turn left with probability $p = 0.1$. Is it a better than 50% chance they will make the light? (What assumptions do you make?)

8. Roll a pair of dice. Write out the distribution for the minimum of the two rolls.

9. Let X be a random number chosen from 1 to n with n an odd number ($n = 2k+1$). Find the mean and variance of X.

10. A student gets a summer job at NYPIRG going door to door. They are told that NYPIRG people average 150 dollars a day with a standard deviation of 25. Knowing this, by Markov's (from class) or Chebyshev's theorem, what is the largest the probability can be of earning more than 300 dollars in a day?

11. Calls come in to a busy math department at an average of 6 per hour. What is the probability that more than 15 will come in during the next hour, assuming a Poisson distribution for the number of calls?

12. Suppose we assume from chapter 5 that $E(X+Y) = E(X)+E(Y)$ etc., where X and Y are random variables. How many rolls of a die will it take *on average* to roll all 6 faces? (Let $X = X_1 + X_2 + X_3 + X_4 + X_5 + X_6$ where X_i is the time it takes to roll a new face, after $i-1$ different faces have been rolled.)

13. A 5-card poker hand can be viewed as sampling without replacement from a 52 element "urn." The hypergeometric distribution can be handy.

    (a) What is the expected number of aces in a 5-card hand?

    (b) What is the probability of 3 or 4 aces?