311 review sheet. The exam covers sections 3.4, 3.5, 3.6, 3.7, 3.8, 4.1, 4.2, 4.3, 4.4, and 4.6 from the text.

Sections 4.1 and 4.2 cover general theory about continuous random variables. Otherwise the material covers some named distributions: the geometric, the negative binomial, the Poisson, the hypergeometric, the uniform, the exponential and the normal. For these you should write down on your study card the p.d.f. (either P(X = k) or the density), E(X) and VAR(X), as these may prove useful.

These are some problems to work one to help prepare you for the second exam in MTH 311. They are not meant to be templates for actual exam questions.

- 1. Roll a die keeping track of the different rolls with $X_1, X_2, ...$ Answer the following questions, using formulas for the named distributions above:
 - (a) In the first 4 rolls, find the probability of rolling no sixes.
 - (b) Find the expected number of rolls until the first six appears
 - (c) Find the expected number of rolls until the 4th six appears.
 - (d) Find the probability that the 4th six appears on the 12th roll (give an expression only).
- 2. A modem connection passes bits of information across a phone line. Suppose the probability a given bit will be corrupted is 10^{-6} and that each bit transferred is independent. What is the probability that no bits are corrupted in the first $5 * 10^{6}$ exchanges?
- 3. A lottery has 80 balls. You choose 10 numbers. Then 20 balls are chosen at random without replacement. Let X be the number of your selections that match any of the chosen balls.
 - (a) What is the range of X?
 - (b) Find the probability that X = 0 and compare to the probability that X = 1 or 2. Which is greater?
- 4. One friend is assigned to roll a die until six 6's appear. Let T_1 be that time. Another friend is assigne to roll two dice until one double six appears. Let T_2 be that time
 - (a) On average, which one will take longer, T_1 or T_2 ?
 - (b) Which value is greater $P(T_1 = 36)$ or $P(T_2 = 36)$?
- 5. Roll a die until a six appears. Let T indicate the number of rolls it takes. Which is greater P(T > 4) or P(T > 10|T > 6)?
- 6. Earthquakes of magnitude 2 or greater happen *on average* two times per year in a given site. Assume that the occurrence of an earthquake during a time period is independent of any occurrence in any other non-overlapping time period and that the rate of earthquakes is constant.
 - (a) Find the expected number of earthquakes in 2 years
 - (b) Find the probability of 0 earthquakes in 2 years.

- 7. Let f(x) = cx(1-x) for x in [0,1] and 0 otherwise. Let X be a random variable with this density.
 - (a) Find c
 - (b) Find P(X > 1/2)
 - (c) Find $P(X \le 1/4)$.
 - (d) Find E(X)
 - (e) Find VAR(X).
- 8. Suppose X is uniform over the interval [10, 15]. Find VAR(X).
- 9. 10 random numbers are chosen uniformly from [0,1] by a computer. Find the probability that 3 or more are greater than 0.75.
- 10. Let X be exponential with mean 2.
 - (a) Find P(X > 2).
 - (b) A random number generator prints out realizations from the exponential distribution with mean 2. On average, how long will you wait (how many numbers to print) before it prints a 4? A 10?
- 11. Let X be normal with $\mu = 20$ and $\sigma = 200$.
 - (a) Find the expected value of 2X 15
 - (b) Find the probability that X < 100.
 - (c) Find the probability that X > 200.
 - (d) Find a value b with P(X > b) = 0.25
 - (e) A process generates realizations of X in an independent manner (like a random number generator). How long will you wait on average before seeing a number between 300 and 400?
- 12. A biologists is studying a population of needle fish and reports that the mean length is 25.4 cm with a 3.1 cm standard deviation. If a needle fish is randomly caught
 - (a) what is the probability that it is more than 30 cms in length?
 - (b) If 4 fish are caught, independently, what is the probability that atleast 1 is more than 30cms in length?