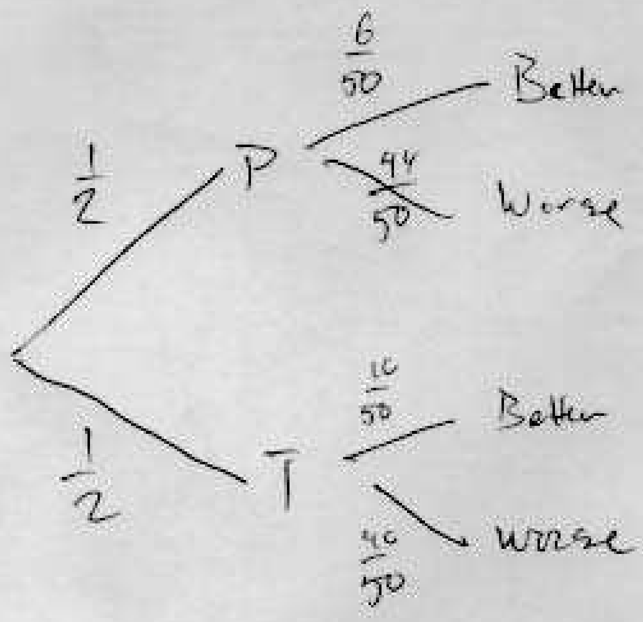
(1)



a) $\frac{56}{100}$

b) $\frac{6+10}{100}$

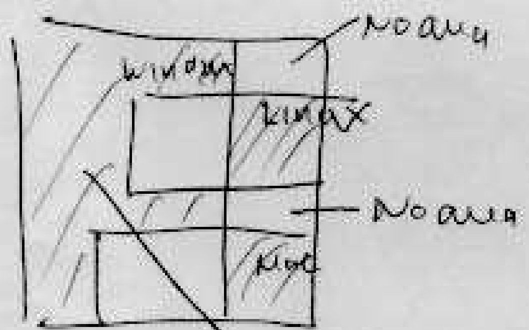
Bayes Thm

c) $\boxed{P[T|B] = \frac{P[T\cap B]}{P[B]}}$

$= \dfrac{\frac{1}{2}\times\frac{10}{50}}{\frac{1}{2}\times\frac{10}{50} + \frac{1}{2}\times\frac{6}{50}}$

d) $\frac{10}{50}$

(2).



b) $P[W \text{ or } M] = P[W] + P[M] - P[W\cap M]$

$= .95 + .10 - .08$

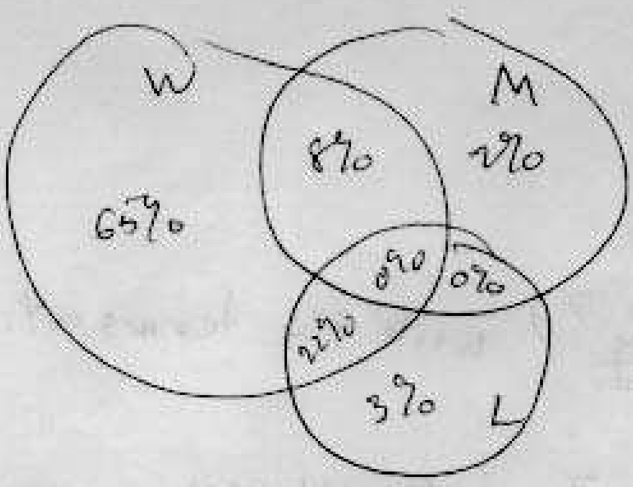
c) $P[\text{only } L] = \cancel{P[L\cap W^c] + P[L\cap M^c]}$

$= P[L\cap (W\cup M)^c]$

$= P[L\cap W^c \cap M^c] =$

Better off figuring out diagram, then answering questions

Try this



since $M \cap L = \phi$

$W \cap M \cap L = \phi$.

So $W \cap L$ has 22%

sum of disjoint events

b) $P[W \text{ or } M] = .65 + .22 + .8 + .0 + .02$.

c) $P[\text{only } L] = .03$

d) $P[\text{all } 3] \le P[M \cap L] = 0$

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(8) $P(A) = .6 \quad P(B) = .3$

a) $P[A \cup B] = P[A] + P[B] - P[A \cap B]$

$= .6 + .3 - 0$ (disjoint)

b) $P[A \cup B] = .6 + .3 - .6 \times .3$ (independent)

c) smallest - make $P[A \cap B]$ the largest. when $B \subseteq A$

$P[A \cup B] \ge .6 + .3 - .3 = .6 = P[A]$.

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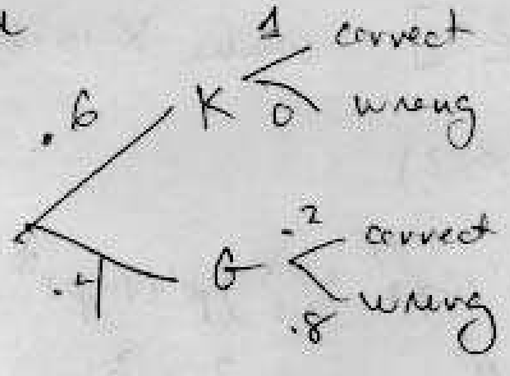
(9) From Birthday Problem

$P[\text{no birthdays match}] = \dfrac{{}_{365}P_k}{365^k}$

in this case

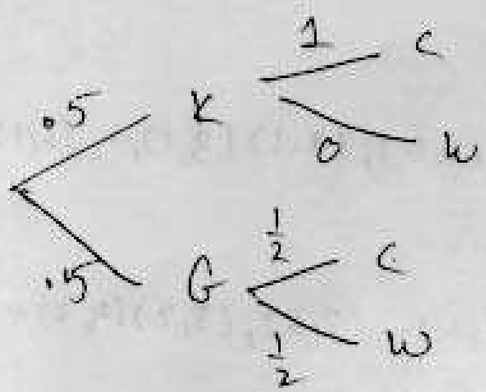
$$\frac{{}_{10^8}P_{10^3}}{(10^8)^{10^3}}$$

(5) a) need tree



$p[\text{correct}] = .6 \times 1 + .4 \times .2$

b)



$P[\text{correct}] = .5 \times 1 + .5 \times .5$

(6) Same as $X \geq 3$ where p X is Binomial$(5, p)$

The key observation to make this a familiar problem

$P[\text{team A wins}] = P[X \geq 3]$

$= P[X=3] + P[X=4] + P[X=5]$

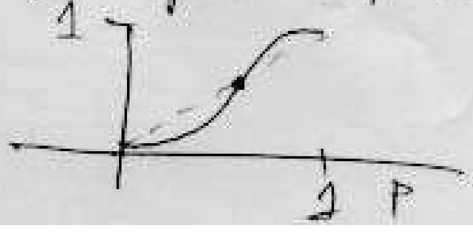
$= \binom{5}{3} p^3 (1-p)^2 + \binom{5}{4} p^4 (1-p)^1 + \binom{5}{5} p^5$

$= 10p^3(1-p)^2 + 5p^4(1-p) + p^5$

$= p^3\left[10(1-p)^2 + 5p(1-p) + p^2\right]$

$= p^3\left[10 - 15p + 6p^2\right]$

use computer to graph p vs $p^3[10 - 15p + 6p^2]$.



7. $P[\text{wake light}] = P[X \le 2]$ where X is binomial $(n=5, p=.1)$

$= P(X=0) + P(X=1) + P(X=2)$

$= \binom{5}{0} p^0 (1-p)^5 + \binom{5}{1} p^1 (1-p)^4 + \binom{5}{2} p^2 (1-p)^3$

$= (.9)^5 + 5(.1)(.9)^4 + 10(.1)^2(.9)^3$

8. Minimum of

$X=1$ $\{(1,1),(1,2) \cdots (1,6),(2,1)(3,1), \cdots (6,1)\}$

$P(X=1) = \frac{11}{36}$

$X=2 = \{(2,2)(2,3)(2,4) \cdots (2,6),(3,2)(4,2) \cdots (6,2)\}$

$P(X=2) = 9/36$

$X=3 = \{(3,3),(3,4) \cdots (3,6),(4,3) \cdots (6,3)\}$

$P(X=3) = 7/36$

$X=4 = \{(4,4),(4,5),(4,6),(5,4),(6,4)\}$

$P(X=4) = 5/36$

$X=5 = \{(5,5),(6,5),(5,6)\}$

$P(X=5) = 3/36$

$X=6 = \{(6,6)\}$

$P(X=6) = 1/36$

9. $\mu = \sum_{i=1}^{n} i \times \frac{1}{n} = \frac{n(n+1)}{2} \cdot \frac{1}{n} = \frac{n+1}{2} = \frac{2k+1+1}{2} = \underline{k+1}$

9. $\sigma^2 = \sum (i-\mu)^2 \cdot \frac{1}{n}$

$= \sum_{i=1}^{2k+1} (i-(k+1))^2 \cdot \frac{1}{n}$

say $n=7, k=3$ Then

$\sigma^2 = \frac{1}{7}\left[(-3)^2+(-2)^2+(-1)^2+0^2+1^2+2^2+3^2\right]$

$= \frac{1}{7} \times 2 \times \left[1^2+2^2+3^2\right]$

$= \frac{1}{n} \times \sum_{j=0}^{k} j^2 \times 2$ (why?) write the terms out.

$= \frac{2}{n} \times \frac{k(k+1)(2k+1)}{6} = \frac{2}{2k+1} \cdot \frac{k(k+1)(2k+1)}{6} = \frac{k(k+1)}{3}$

10 $P[X > 300] \leq \frac{E(X)}{300}$ by Markov

$= \frac{150}{300} = \frac{1}{2}$.

where

$\frac{1}{k^2} \geq P[|X-\mu| \geq k\sigma] \geq P[X-\mu > k\sigma]$

$= P[X-(\mu+k\sigma)>0]$.

$= P[X \geq \mu+k\sigma]$

So if $300 = \mu+k\sigma$

$= 150 + k\times 25$

$\frac{300-150}{25} = 6 = k$ and so

$P[X \geq 300] \leq \frac{1}{k^2} = \frac{1}{36}$ By Chebyshev.