The final exam is comprehensive, although not all parts will be equally represented. Here are some practice problems for the new material. Some are from Monday's lecture. You are allowed **three** 3 by 5 cards for formulas. The test will be Monday during the regular class time. I will give an extra half hour after the exam if needed.

1. A store manager views each customers bill as an independent sample from a common distribution with mean \$25 and standard deviation \$15. Let $S = X_1 + X_2 + \cdots + X_{16}$ be the amount spent by 16 customers.

   (a) What is the expected value of S? What is the standard deviation?

   (b) Using Chebyshev's theorem estimate
   $$P(S \geq 450).$$

   (c) Using the central limit theorem (assuming n is large enough) estimate
   $$P(S \geq 450).$$

   (d) How would your last answer change if you were told that the population for each customer's bill is normally distributed?

2. A recent news article has

   > The world's population is aging and fattening, fueling a continued increase in blood pressure problems. Remarkably, the report cites worse hypertension rates in much of Western Europe than in the U.S., despite cultural similarities: 38 percent in England, Sweden and Italy; 45 percent in Spain; 55 percent in Germany.

   To assess the blood pressure rate in the United States, 1000 patients were randomly selected. It was found the 350 had high-blood pressure. Find a 95% confidence interval for the true percentage for the US. Does it contain 38%?

3. In the simple linear regression model from statistics there are two variables, say X and Y. The value of Y depends on X, typically by assuming the mean of Y depends conditionally on the value of X.

   Suppose the joint distribution of X and Y is uniform over the region described by
   $$0 \leq x \leq 2; \quad -1 + x \leq y \leq 1 + x$$

   (a) Find $f(x, y)$.

   (b) Find the marginal distribution of X.

   (c) Find $f(y|x)$ the conditional density of y given x.

   (d) Find $\mu_{y|X=x}$, the conditional mean of Y given the $X = x$.

4. The sample standard deviation
   $$S^2 = \frac{1}{n-1}[\sum X_i^2 - \frac{1}{n}(\sum X_i)2]$$
   has $n-1$ instead of n in the denominator to make it unbiased. That is $E(S^2) = \sigma^2$.

   Verify this for a random sample when $n = 2$ by computing $E(S^2)$

5. The tire pressure of the left and right front tires is modeled using random variables X and Y. A tire is properly filled if it has 26 pounds of pressure. Suppose X and Y have joint density

$$f(x,y) = K(x^2 + y^2) \quad 20 \le x, y \le 30$$

and 0 otherwise.

   (a) Find K
   (b) What is the probability both tires are underfilled?
   (c) What is the probability that the difference between X and Y is no more than 2 pounts ($P(|X - Y| < 2)$).
   (d) Find the marginal distribution of X. (It will also be the marginal distribution of Y).
   (e) Are X and Y independent?

6. Suppose the joint density of X and Y is given by

$$f(x,y) = 24xy, \quad 0 \le x \le 1, 0 \le y \le 1, x + y \le 1,$$

and 0 otherwise.

   (a) Compute $f_X(x)$ which will also be $f_Y(y)$.
   (b) Find $E(X)$ and $E(Y)$.
   (c) Compute the covariance of X and Y, $E(XY) - E(X)E(Y)$.

7. A gas station has 2 self-serve pumps and 2 full-serve pumps. At any given time, they figure the number of pumps being used is given by

		Self Serve		
		0	1	2
Full	0	.10	.04	.02
Serve	1	.08	.20	.06
	2	.06	.14	.39

Let X be the number of full serve pumps and Y the number of Self Serve pumps in use.

   (a) What is $P(X = 1, Y = 1)$?
   (b) Compute $P(X \le 1, Y \le 1)$
   (c) Describe in words, then find the probability of the event $\{X \ne 0, Y \ne 0\}$.
   (d) What is $P(X \le 1)$?
   (e) Are X and Y independent?
   (f) Find $E(X + Y)$, the expected number of pumps in use.

8. A student is asked to select 5 points uniformly from the unit box: $0 \le x \le 1, 0 \le y \le 1$. What is the probability that exactly 4 points have coordinates with both x and y bigger than $1/2$?