Test 2 covers the following topics:

The binomial distribution This describes the number of successes in n independent trials

The normal distribution The bell shaped curve

The Central Limit Theorem This theorem describes the distribution of the sample mean $\overline{x}$ as n, the sample size, gets large.

Sampling distributions In general, a statistic summarizes a random sample an consequently is a random variable. A statistics distribution is known as a sampling distribution. We only spoke about $\hat{p}$ and $\overline{x}$, but the topic is more general.

Confidence intervals for p given $\hat{p}$ Recall the $(1-\alpha)\cdot 100\%$ confidence interval for p given $\hat{p}$ is

$$\hat{p} - a\mathsf{SE}(\hat{p}) \le p \le \hat{p} + a\mathsf{SE}(\hat{p}),$$

where a is related to α by the normal distribution: $P(-a \le Z \le a) = 1-\alpha$, or $P(Z \ge a) = \alpha/2$.

Some sample problems I might ask would be: [Answers follow, using the R commands `pnorm` to replace the $P(Z \le z)$ and `dbinom(n,k)` to compute $P(X = k)$ for a binomial X]

1. If X is binomial with $n = 5$ and $p = 1/4$ find all of the following:

$$E(X), \quad \mathsf{SD}(X), \quad P(X = 3), \quad P(X \le 1)$$

Answer:
We have

```
> n = 5; p = 1/4
> EX = n * p; EX

[1] 1.25

> SDX = sqrt(n * p * (1 - p))
> SDX

[1] 0.9682

> PX3 = dbinom(3, n, p)
> PX3

[1] 0.08789
```

```
> PX1 = dbinom(1, n, p) + dbinom(0, n, p)
> PX1

[1] 0.6328
```

2. Suppose X is binomial with $n = 4$ and $p = 1/2$ and Y is binomial with $n = 6$ and $p = 1/3$. Which is more likely $P(X = 2)$or $P(Y = 2)$?

Answer:
We have the same expectation ($np = 2$ for each, but for probabilities

```
> dbinom(2, 4, 1/2)

[1] 0.375

> dbinom(2, 6, 1/3)

[1] 0.3292
```

Showing $P(X = 2)$ is larger.

3. To see how effective text-messaging is for contacted students, 100 text messages were sent to 100 randomly chosen students. If the probability of being read is $p = .75$ compute the expected number read. Find the z score for 80 being read.

Answer:
The expected number for a binomial is $np = 75$. The z-score is found by $(x - mean)/sd$ or

```
> (80 - 75)/sqrt(100 * 0.75 * (1 - 0.75))

[1] 1.155
```

4. Let Z be a standard normal Find the following:

$$P(Z < 1), \quad P(Z \leq 2.3), \quad P(Z \geq 1.23), \quad P(-1 \leq Z \leq 1/2)$$

Answer:
We use `pnorm` here instead of a table. You'd use a table on the test.

```
> pnorm(1)

[1] 0.8413

> pnorm(2.3)

[1] 0.9893

> 1 - pnorm(1.23)

[1] 0.1093

> pnorm(1/2) - pnorm(-1)

[1] 0.5328
```

5. Again, let Z be a standard normal. Find z for each

$$P(Z \leq z) = .32, \quad P(Z \geq z) = 0.10$$

Answer:
These questions are about quantiles. We need to use `qnorm()` here as we don't have a table.

```
> qnorm(0.32)

[1] -0.4677

> qnorm(1 - 0.1)

[1] 1.282
```

6. Let Y be a normal random variable with mean 10 and standard deviation 20. Find

$$P(Y > 10), \quad P(Y > 20), \quad P(Y > 31), \quad P(15 < Y < 25)$$

Answer:
We find the z-score, then use `pnorm()`.

```
> 1 - pnorm((10 - 10)/20)

[1] 0.5

> 1 - pnorm((20 - 10)/20)

[1] 0.3085

> 1 - pnorm((31 - 10)/20)

[1] 0.1469

> pnorm((25 - 10)/20) - pnorm((15 - 10)/20)

[1] 0.1747
```

7. Let X_1, X_2, ..., X_{16} is random sample for a normal population with mean 10 and standard deviation 20. Find the following

$$P(\bar{x} > 10), \quad P(\bar{x} > 20), \quad P(\bar{x} > 31), \quad P(15 < \bar{x} < 25)$$

Answer:
Why is this different from the previous? Because the random variable is $\bar{x}$. The z scores need to be computed using $\bar{x}$'s mean and standard deviation. The mean is still μ, but the standard deviation is now $\sigma/\sqrt{25}$.

```
> 1 - pnorm((10 - 10)/5)

[1] 0.5

> 1 - pnorm((20 - 10)/5)

[1] 0.02275

> 1 - pnorm((31 - 10)/5)

[1] 1.335e-05

> pnorm((25 - 10)/5) - pnorm((15 - 10)/5)

[1] 0.1573
```

8. Suppose waist sizes are normally distributed with a mean of 92 cm and standard deviation of 11cm. Let Y denote a randomly chosen waist, find

   (a) $P(Y \geq 100)$.

   (b) $P(Y \geq y) = 0.80$

   Answer:
   The first is done with a z-score

```
> 1 - pnorm((100 - 92)/11)

[1] 0.2335
```

   The second is a quantile problem. First we find the z score of the quantile, then we convert into the right scale

```
> a = qnorm(1 - 0.8)
> 92 + a * 11

[1] 82.74
```

9. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from a population with mean μ and standard deviation σ. Which of these statements actually makes sense?

   (a) The sample mean is the population mean.

   (b) The mean of the sample mean is the population mean.

   (c) the standard deviation of the sample mean is the population standard deviation.

   (d) The distribution of the sample mean (for large n) is not the population distribution but the normal distribution.

   Answer:
   The first is false – the sample mean is random, not a constant. The second is true. The third is false: it is $\sigma/\sqrt{n}$. Finally, the fourth is true by the Central Limit Theorem.

10. A survey of 365 Connecticut residents found a 60% supported current Senator Joe Lieberman. Find a 90% confidence interval for the population proportion.

Answer:
We first find a using `qnorm()`. (Why do we use 0.95?)

```
> a = qnorm(0.95)
> a

[1] 1.645

> SE = sqrt(0.6 * (1 - 0.6)/365)
> SE

[1] 0.02564

> MOE = a * SE; MOE

[1] 0.04218

> c(0.6 - MOE, 0.6 + MOE)

[1] 0.5578 0.6422
```

11. A random sample of 50 people finds 46% agree with some proposition.

    What is the margin of error?

    Is .5 in the 95% CI based for π based on this sample?

    How large would n need to be so that if $\widehat{p} = .46$ you would be sure that $p = .5$ is not in the 95% CI for p given by $\widehat{p}$. (Set the margin of error to 0.04)

Answer:
The margin of error depends on the confidence level. We'll use 95% so that we have

```
> SE = sqrt(0.46 * (1 - 0.46)/50)
> MOE = 1.96 * SE
> MOE

[1] 0.1381
```

Clearly 0.5 would be in a confidence interval. The margin of error informs us that 95% of the time $\widehat{p}$ within a margin of error of p.

If the MOE was less than 4 percentage points, then 0.5 would not be in the confidence interval. Using a formula from the notes, this will be the case is n is atleast as large as

```
> a = 1.96; MOE = 0.04
> (a/MOE)^2 * (1/4)
```

```
[1] 600.2
```

12. The confidence interval for p based on $\hat{p}$ requires a random sample from the population. Explain why this isn't the case in the following scenarios:

    (a) The population is all US teens. The sample contains 1200 myspace users.

    (b) The population is all people with colds. The sample contained only people who used homeopathic remedies.

    (c) The population is all US teens. The sample is made up of the 120 friends listed by a myspace user.

Answer:

Myspace users are not a random sample of US teens, presumably, so this population would introduce a bias into any calculation.

People who use homeopathic remedies do not necessarily represent the entire population of people who have colds.

A persons friends from myspace are in no way a random sample. An old adage "Birds of a feather flock together" might be interpreted in this case as saying the indivual sampling units are not independent.