

This is a review for test number 2 which occurs Monday March 29.

We cover the material from 2/23 on probability to 3/22 on the binomial. The major topics were

Probabilities We learned how to compute the probability of an event for *equally likely outcomes*. This was

$$P(E) = \frac{\text{The number of outcomes in } E}{\text{total number of outcomes}}.$$

The way to compute probabilities then became a question of counting the top and bottom.

Counting We learned how to count using the following formulas

number of possibilities = $n_1 \cdot n_2 \cdots n_k$ multiplication rule

$${}_nP_r = n \cdot (n-1) \cdots (n-r+1) = \frac{n!}{(n-r)!} \quad \text{number of permutations of length } r$$

$${}_nC_r = \binom{n}{r} = \frac{n!}{(n-r)!r!} \quad \text{number of combinations of length } r$$

The multiplication rule is the general purpose one. The permutations formula is a special consequence where each stage has one fewer possible choice. The rule for combinations is also read “ n choose r ” which is how it is often used – choosing some subset of the entire n objects.

Random Variables We talk about **random variable**, their **distribution**, their **mean** and their **standard deviation**. The distribution allows us to specify probabilities more generally than the case when all outcomes are equally likely. It requires $\sum P(X = k) = 1$. The mean and standard deviation are computations that you need to be able to make.

$$\mu = E(X) = \sum_k kP(X = k)$$

$$\sigma^2 = \sum_k (k - \mu)^2 P(X = k)$$

Make these with a table and they become easy to do.

Binomial distribution This is a special distribution for us. We learned a formula for its distribution

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}.$$

It has mean $\mu = np$ and standard deviation $\sigma = \sqrt{np(1-p)}$. This comes about by counting the number of successes of some repeated process.

When approaching a binomial problem, always identify the two parameters n and p , and then write the question in terms of a probability statement like $P(X \leq k)$.

Normal distribution This is a continuous distribution for random numbers. The probabilities are specified in terms of areas of a picture. This picture depends on two parameters μ and σ , although we get the actual numbers using the standard normal and the z scores.

When doing a normal problem, you need to identify the two parameters μ and σ . Then draw a picture using area to indicate the desired probability. From here, you usually have to work the problem down to one involving the special areas answered by the table in the book. A key is the symmetry of the normal curve, and the fact that the total area is 1.

Normal approximation to the binomial The probabilities for a binomial can be hard to compute. There is an approximation though given by the normal distribution. In short, the normal distribution with mean μ and standard deviation σ given by the mean and standard deviation of the binomial, np , and $\sqrt{np(1-p)}$.

The approximation says

$$P(X \leq b) \approx P(Z \leq \frac{b + 1/2 - np}{\sqrt{np(1-p)}})$$

$$P(a \leq X) \approx P(Z \geq \frac{a - 1/2 - np}{\sqrt{np(1-p)}})$$

The extra 1/2 is the continuity correction and only makes sense if you draw the picture with binomial probabilities written as areas of rectangles.

Some sample problems are given below. These are representative of questions that may appear on the exam, but do not think that all the exam questions will look like one of these.

1. Suppose the distribution of X is

k		0	1	2	3	4	5

P(X=k)		1/2	1/4	1/8	1/6	1/32	??

Find the value of $P(X = 5)$, the mean and standard deviation of X .

2. Let X be the number of heads in 4 coin tosses. Find the distribution of X . Use it to answer, $P(X > 2)$.
3. If there are 10 colors for a new car, and you can choose 3 to display on your car lot, how many choices do you have?
4. You need to park 5 cars in a row, how many ways can this be done?
5. A new car carries 3 color choices, two transmission types and 3 engine sizes. If you want one of each on your car lot how many must you have?
6. A manufacturer has 3 SUV's, 2 station wagons and 3 sedans. If you will have 2 of each type on your car lot how many different choices do you have?
7. A car dealer makes \$4000 on an SUV, \$3500 on a luxury car and \$500 on an economy car. If these sell with probability .4, .2, .4 respectively, what is the expected amount made on a car sale?

8. Historically, one of every 15 customers to a car dealership buys a car. If one Saturday 100 people come in what is the expected number of cars sold? What is the probability of exactly 3 cars being sold? Use the normal approximation to the binomial to find the probability that 6 or fewer cars are sold.
9. A person who owns a honda has a probability of .7 of their next car being a Honda. Our of 200 Honda owners what is the expected number whose next car will be a Honda? What is the probability 150 or more will own a Honda with their next car? (Use the normal approximation)
10. The price range of a new car sold is assumed to be normal with mean 20,000 and standard deviation 4,000. What is the probability that a new car will cost 25,000 or less? What value do 95% of cars sell for less than?
11. The amount of time a new tire will last is normally distributed with mean 40,000 miles and standard deviation 7,500 miles. What is the probability a new tire lasts more than 50,000 miles? How many miles do at least 80% of new tires get?