Some key things:

For points $(x_0,y_0),\ldots,(x_n,y_n)$ where $y_i = f(x_i)$, there is a unique polynomial of degree n or less with $p(x_i) = y_i$. This polynomial may be written as

$$p(x) = f[x_0] + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1) + \cdots + f[x_0,x_1,\ldots,x_n](x-x_0)\cdots(x-x_{n-1}), \text{ or}$$

$$p(x) = \sum_{i=0}^{n} f(x_i) \prod_{j=0, j\neq i}^{n} \frac{x-x_j}{x_i-x_j}.$$

We had two formulas to approximate the derivative at a point:

$$\text{forward difference: } f'(x) \approx \frac{f(x+h)-f(x)}{h},$$

$$\text{central difference: } f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}.$$

For both, there is a balance needed between truncation error and floating point error.

We discussed three types of integration using interpolating polynomials based on equally spaced points: Riemann, trapezoid, Simpsons. Applied to a simple interval these give the following approximations to $\int_a^b f(x)dx$: $f(a)(b-a)$, $(1/2)(b-a)(f(a)+f(b))$, and $(1/6)(b-a)(f(a)+4f((a+b)/2)+f(b))$. The error when applying these formulas in a composite manner are $O(h)$, $O(h^2)$, and $O(h^4)$ respectively.

We discussed quadrature formulas, such as Gauss quadrature where $\int_{-1}^{1} f(x)dx$ is approximated by a sum $\sum f(x_i)A_i$ where x_i are *nodes* and A_i are *weights*. By choosing nodes appropriately we can ensure that using $n+1$ points yields a method that exactly integrates any polynomial of degree $2n+1$.

For initial-value problems we discussed some theorems that guarantee convergence: if both f and $\partial f/\partial x$ are *continuous* in R, then there is a unique solution for $|t-t_0| \leq min(\alpha, \beta/M)$; and If f is Lipshitz then the intial value problem will have a unique solution in some interval.

We discussed various schemes for approximating solutionsn to initial value problems. In particular:

- Euler's method: $x_{n+1} = x_n + hf(t_n,x_n)$
- Backwards Euler: $x_{n+1} = x_n + hf(t_{n+1},x_{n+1})$
- Runge Kutte including:
  - Heun: $x_{n+1} = x_n + (1/2)F_1 + (1/2)F_2, \quad F_1 = hf(t_n,x_n), \quad F_2 = hf(t_n+h/2, x_n+F_1/2)$
  - RK45: $x_{n+1} = x_n + (1/6)(F_1+4F_2+4F_3+F_4)$, where $F_3 = hf(t_n+h/2, x_n+F_2/2)$, $F_4 = hf(t_n+h, x_n+F_3)$
- Multistep including:

- Second order Adams-Bashworth: $x_{n+1} = x_n + h[(3/2)f_n - (1/2)f_{n-1}]$
- Second order Adams-Moulton: $x_{n+1} = x_n + h[(5/12)f_{n+1} + (2/3)f_n - (1/12)f_{n-1}]$

For multistep models we discussed that the algorithms are convergent if and only if they are stable and consistent. The latter two conditions can be checked by related polynomials.

For multistep models with $f_x \leq \lambda < \infty$ we discussed that if the local truncation error was $O(h^{m+1})$ the global truncation error was $O(h^m)$.