This is a timed exam with only the class time to complete it.

A quick review of the notes is:

- Newton's method: $x_{n+1} = x_n - f(x_n)/f'(x_n)$; $e_{n+1} \approx f''(r)/(2f'(r))e_n^2$.
- Secant method: $x_{n+1} = x_n - f(x_n)\cdot(x_n - x_{n-1})/(f(x_n) - f(x_{n-1}))$; $e_{n+1} \approx f''(r)/(2f'(r))e_n e_{n-1}$.
- Fixed point method: $x_{n+1} = F(x_n)$, $e_{n+1} \approx F^{(q)}(r)/q!e_n^q$ where q is the smallest integer with $F^{(q)}(r) \neq 0$.

For $Ax = b$ we have these facts about square A:

- If all n leading principal minors of A are non-singular, then A has an LU decomposition (without pivoting).
- If A is real, symmetric and positive definite then it has a unique (Cholesky) factorization $A = LL^T$ and L has a positive diagonal.
- With pivoting, there is L, U, and P with $LU = PA$.
- Solving $Ax = b$ with Gaussian elimination and scaled pivoting for m vectors b takes approximately $n^3/3 + (1/2+m)n^2$ ops.

A vector norm induces a matrix norm. In particular:

- The l_2 vector norm (Euclidean distance) induces $\|A\|_2$ defined by the largest singular value (in magnitude) of A.
- The l_∞ vector norm (largest component) induces $\|A\|_\infty$ defined by the largest l_1 norm of the rows. (The l_1 vector norm is the sum of the absolute value of the components.)

The relationship between relative errors in b and x depends on the condition number of A, $\kappa(A) = \|A\| \cdot \|A^{-1}\|$, and is given by:

$$\frac{\|x - \tilde{x}\|}{\|x\|} \leq \kappa(A)\frac{\|b - \tilde{b}\|}{\|b\|}$$

The general iteration scheme $x^{(n+1)} = Gx^{(n)} + c$ will converge to $x = (I-G)^{-1}c$ if and only if the spectral radius of G is less than 1. A special case of this, when Q is a splitting matrix of A ($A = Q + B$ and Q is invertible). In that case $G = Q^{-1}(Q - A)$ and $c = Q^{-1}b$.