1 Questions to be handed in for project 4:

Read about this topic here: Solving for zeros with julia.

When done with this project, you can print it by *first* selecting the parts to print, the printing the selection.

1.1 Overview

These questions are about finding zeros using `julia`. A zero of a function $f(x)$ is a value x with $f(x) = 0$. There are some types of functions where these can always be found, for example linear functions and quadratic functions. For general polynomials, the `roots` function can be used to find all the zeros of a polynomial, algorithmically.

For many other functions it is not an automatic process to find all zeros. Rather we discuss a two-step process to identify a given zero:

- identify graphically a desired zero. We need *simple zeros* where the function actually crosses the x axis and doesn't just touch it. A *bracket* is an interval $[a, b]$ where $f(a)$ and $f(b)$ have different signs, so by the intermediate value theorem, if $f(x)$ is continuous it will have at least one zero in the interval.
- Rather than graphically zoom in on the identified zero, we use an algorithm to then locate the zero. The notes show how to implement the bisection algorithm and the function `fzero` from the `Roots` package implements a faster algorithm. Both are used to locate a zero within a bracket and have an identical interface.

To get started, we will use three packages. Load these and ignore the warnings:

```
using Gadfly            # ignore any warnings
using Polynomial
using Roots
```

1.2 Questions to answer

- Find a zero of the function $f(x) = 216 - 0.65x$.
- The parabola $f(x) = -16x^2 + 200x$ has one zero at $x = 0$. Graphically find the other one.
- The polynomial $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is represented in `julia` with this: `Poly([an, an_1, ..., a1, a0]` For example, if $p(x) = 1x^3 - 1x^2 - 14x + 24$, then the polynomial and its roots can be found with:

```
p = Poly([1, -1, -14, 24])
roots(p)
```

Use the `roots` function to find the zeros of $p(x) = x^3 - 4x^2 - 7x + 10$.

- Use the `roots` function to find the *real* zeros of $p(x) = x^5 - 5x^4 - 2x^3 + 13x^2 - 17x + 10$. (There are 5 zeros, not all real.)

- Let $p(x) = x^4 - 7x^3 + 5x^2 + 31x - 30$. The `polydir` function gives a related polynomial, call it $q(x)$. What is the relationship between the zeros of p and those of q?

```
p = Poly([1, -7, 5, 31, -30])
q = polydir(p)
```

You can answer this by comparing the roots, but this graph might be more helpful in answering (the `polyval` function is used to evaluate the polynomial at a value `x`):

```
plot([x -> polyval(p, x), x -> polyval(q,x)], -2.25, 5.5)
```

- Graph the function $f(x) = x^2 - 2^x$. Try to graphically estimate all the zeros. Write answers to one decimal point.

- Plot the conversion of Celsius to Fahrenheit ($f(x) = 9/5x + 32$) and the line $y = x$. The intersection point is the temperature where both systems give the same value. What is the value?

- Find the point(s) of intersection of the graphs of $f(x) = 2.5 - 2e^{-x}$ and $g(x) = 1 + x^2$.

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The notes have a `bisection` method, here is an abbreviated version.

```
function bisection(f, bracket)
  a,b = bracket
  mid = (a+b)/2

  while a < mid < b
    if f(mid) == 0.0 break end
    f(a) * f(mid) < 0 ? (b = mid) : (a = mid)
    mid = (a+b) / 2
  end
  mid
end
```

Read the notes to understand the logic above. This function is used to find a zero, when $[a, b]$ bracket a zero for f. It is called like `bisection(f, [a,b])`, for a suitable `f`.

Use this function to find a zero of $f(x) = \sin(x)$ on $[3, 4]$:

- Let $f(x) = \exp(x) - x^5$. In the long run the exponential dominates the polynomial and this function grows unbounded. By graphing over the interval $[0, 15]$ you can see that the largest zero is less than 15. Find a bracket and then use `bisection` to identify the value of the largest zero.

- The `Roots` package has a built-in function `fzero` that does different things, with one of them being a (faster) replacement for the `bisection` function. That is, if `f` is a continuous function and `[a,b]` a bracketing interval, then `fzero(f, [a,b])` will find a zero of `f`.

Show that it works by finding a zero of the function `f(x) = (1 + (1 - n)^2)*x - (1 - n*x)^2` when $n = 10$. Use $[0, 0.5]$ as a bracketing interval.

- The `airy` function is a special function of historical importance. Find its largest negative zero by first plotting, then finding a bracketing interval and finally using `fzero` to get a numeric value.

- Suppose a crisis manager models the number of cases of water left after x days by $f(x) = 550,000 \cdot (1 - 0.25)^x$. When does the supply of water dip below $100,000$? Find a bracket and then use a numeric method to get a precise answer.