1 Questions to be handed in for project 5:

Read about this material here: Investigating limits with julia.

When done with this project, you can print it by *first* selecting the parts to print, the printing the selection.

To get started, we load `Gadfly` so that we can make plots.

```
using Gadfly            # ignore any warnings
```

1.0.1 Quick background

We can investigate limits three ways: analytically, with a table of numbers, or graphically. Doing so analytically is for the classroom or a symbolic math program. Here we discuss two ways: graphically or numerically.

Investigating a limit numerically requires us to operationalize the idea of x getting close to c and $f(x)$ getting close to L. The first is easy: just create numbers getting close to 0:

```
hs = [(1/10)^i for i in 1:10]
```

Then we can investigate limits by looking the corresponding `f(x)` values. For example, the limit of $\sin(x)/x$ near 0 is investigated with:

```
f(x) = sin(x)/x
ys = [f(x) for x in hs]     # y values. Alternatively ys = map(f, hs)
[hs ys]              # arrange in a table
```

From this we see a *right* limit at 0 appears to be 1.

Graphically we can see both the right and left limit is 1:

```
plot(f, -1, 1)
```

From the graph, we see clearly that as x is close to 0, $f(x)$ is close to 1. (Gadfly either doesn't include a point for `0` or ignores that value when present by treating it like `NaN`.)

1.0.2 Questions

- Find the limit using a table. What is the estimated value?

$$\lim_{x \to 0+} \frac{\cos(x) - 1}{x}.$$

- Find the limit using a table. What is the estimated value?

$$\lim_{x \to 0+} \frac{\sin(5x)}{x}.$$

- Find the limit using a table. What is the estimated value? (You need values getting close to 3, these can be `3 + hs` or `3 - hs`.)

$$\lim_{x\to 3} \frac{x^3 - 2x^2 - 9}{x^2 - 2x - 3}.$$

- Find the limit using a table. What is the estimated value?

$$\lim_{x\to 0+} \frac{x - \sin(|x|)}{x^3}.$$

- Find the *left* limit of `f(x) = cos(pi/2*(x - floor(x)))` as x goes to 2. (Use values `2 - hs` to investigate.)

- Find the limit using a table. What is the estimated value? Recall, `atan` and `asin` are the names for the appropriate inverse functions.

$$\lim_{x\to 0+} \frac{\tan^{-1}(x) - 1}{\sin^{-1}(x) - x}.$$

1.0.3 Graphical approach

- Plot the function to estimate the limit. What is the value?

$$\lim_{\theta\to 0} \frac{\sin(5\theta)}{\sin(2\theta)}.$$

- Plot the function to estimate the limit. What is the value?

$$\lim_{\theta\to 0} \frac{2^x - \cos(x)}{x}.$$

- Plot the function to estimate the limit. What is the value?

$$\lim_{\theta\to 0} \frac{\sin^2(4\theta)}{\cos(\theta) - 1}.$$

1.1 Other questions

- Let `f(x;k=1) = sin(sin(x)^2) / x^k`. Consider $k = 1$, 2, and 3. For which of values of k does the limit at 0 **not** exist?

- Let `l(x; a=1) = (a^x - 1)/x` and *define* $L(a) = \lim_{x\to 0} l(x, a)$.

Show that $L(3 \cdot 4) = L(3) + L(4)$ by computing all three limits numerically. (In general, you can show algebraically that $L(a \cdot b) = L(a) + L(b)$ like a logarithm.

1.2 Limits at infinity.

By using x values which grow large, we can get a sense of a limit as x goes to infinity. (This requires a different definition that the ϵ-δ limit.) For example, this suggests the limit of $f(x) = \sin(x)/x$ at infinity is 0:

```
hs = [10.0^i for i in 1:8]
f(x) = sin(x)/x
ys = [f(x) for x in hs]
[hs ys]
```

- Find the limit at infinity of $f(x) = x^5/e^x$ (Exponential eventually grow faster than polynomials).

- Find the limit at infinity of $f(x) = e^t/(1 + e^{-t})$.