# 1 Questions to be handed in for project 7:

Read about this material here: Exploring first and second derivatives with Julia.

When done with this project, you can print it by *first* selecting the parts to print, the printing the selection.

To get started, we load `Gadfly` so that we can make plots, and load the `Roots` package for `D` and `fzero`:

```
using Gadfly           # ignore any warnings
using Roots          # for D and fzero
```

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### 1.0.1 Quick background Read the notes for more detail

This assignment looks at the relationship between a function, $f(x)$, and its first and second derivatives, $f'(x)$ and $f''(x)$. The basic relationship can be summarized (though the devil is in the details) by:

- if the first derivative is *positive* on $(a, b)$ then the function is *increasing* on $(a, b)$.
- If the second derivative is *positive* on $(a, b)$ then the function is *concave up* on $(a, b)$.

Some useful things for `julia` for investigating this are the operator `D` from the `Roots` package which can find the first and second derivatives of `f`. The use follows this pattern:

```
f(x) = sin(x)
fp(x) = D(f)(x)          # makes fp the first derivative
fpp(x) = D(f,2)(x)       # makes fpp the second derivative
```

In the notes, the following function is used to plot a function `f` two ways: once as usual, the second time showing the function `f` *only if* the function `g` is positive.

```
function plotif(f, g, a, b)
  plot([f, x -> g(x) > 0.0 ? f(x) : NaN], a, b)
end
```

This allows a graphical exploration of the above facts:

```
plotif(f, fp, 0, 2pi)       # shows sin(x) and when derivative is 0
```

Clicking on the `f1` in the legend on the right will hide the graph of `f` and leave only the graph of `f` where `fp(x)` $>$ `0`.

### 1.0.2 Questions, Graphical approach

- Make a graph using `plotif` to investigate when the `airy` function is positive on the interval $(-5, 5)$. Your answer should use interval notation.

- Make a graph using `plotif` to investigate when the function $f(x) = x^x$ is *increasing* on the interval $(0, 2)$. Your answer should use interval notation.

- Make graph using `plotif` to investigate when the function

$$f(x) = \frac{x}{x^2 + 9}$$

is *concave up* on the interval $(-10, 10)$. Your answer should use interval notation.

- Make a graph using `plotif` to identify any *critical points* of $f(x) = x\ln(x)$ on the interval $(0, 4)$.

- Make a graph using `plotif` to identify any *inflection points* of $f(x) = \sin(x) - x$ over the interval $(-5, 5)$.

#### 1.0.3 Finding numeric values

- We can graphically identify *critical points* of $f(x)$ by graphing the function's derivative and looking for when the derivative is 0 or undefined. Numerically, we can locate values where the derivative crosses 0 using the `fzero` function from the `Roots` package.

Use `fzero` to numerically identify all *critical points* to the function $f(x) = 2x^3 - 6x^2 - 2x + 4$. (There are no more than 2.)

- Use `fzero` to numerically identify all *inflection points* for the function $f(x) = \ln(x^2 + 2x + 5)$.

- Suppose $f'(x) = x^3 - 6x^2 + 11x - 6$. Where is $f(x)$ increasing? Use interval notation in your answer.

- Suppose $f''(x) = x^2 - 3x + 2$. Where is $f(x)$ concave up? Use interval notation in your answer.

### 1.0.4 the derivative tests

- The first derivative test states that for a differentiable function $f(x)$ with a critical point at $c$ then if $f'(x)$ changes sign from $+$ to $-$ at $c$ then $f(c)$ is a local maximum and if it changes sign from $-$ to $+$ then $f(c)$ is a local maximum.

For the function $f(x)$ suppose you know $f'(x) = x^3 - 5x^2 + 8x - 4$. Find *all* the critical points. Use the first derivative test to classify them as local extrema *if* you can. If you can't say why.

- The second derivative test states that if $c$ is a critical point of $f(x)$ and $f''(c) > 0$ then $f(c)$ is a local minimum and if $f''(c) < 0$ then $f(c)$ is a local maximum.

Suppose $f'(x) = (x^2 - 2) \cdot e^{-x}$. First find the critical points of $f(x)$, then use the second derivative test to classify them.

- For a polynomial $p(x)$ between any two zeros there must be a critical point, and perhaps more than one. Verify this is the case for $p(x) = x^4 + x^3 - 7x^2 - x + 6$. You can do this graphically or numerically.

### 1.0.5 Concave up has alternate definitions

- The book defines $f(x)$ to be concave up for differentiable functions by $f(x)$ is concave up on $(a, b)$ if $f'(x)$ is increasing on $(a, b)$. More generally, one can define $f(x)$ as concave up on $(a, b)$ if for any pair of points $a < c < d < b$ one has the secant line connecting $(c, f(c))$ and $(d, f(d))$ lies *above* the graph of $f(x)$ over $(c, d)$.

For the function $f(x) = x^2 - 2x$, graphically verify this is the case for 3 pairs of points between $(-5, 5)$. The following can be used to create a function for a secant line between $c$ and $d$:

```
function secline(f, c, d)
  x0, y0, m = c, f(c), (f(c) - f(d)) / (c - d)
  x -> y0 + m * (x - x0)    # pt-slope form as function
end
```

- For the function $f(x) = x^3 - 2x$ find a pair of points, $c$ and $d$, in $(-3, 3)$ which illustrate that the function is not concave up.