1 Questions to be handed in on extrema:

Read about this material here: Maximization and minimization with julia.

When done with this project, you can print it by *first* selecting the parts to print, the printing the selection.

To get started, we load `Gadfly` so that we can make plots, and load the `Roots` package for `D` and `fzero`:

```
using Gadfly            # ignore any warnings
using Roots          # for D and fzero
```

1.0.1 Quick background Read the notes for more detail

Extrema is nothing more than a fancy word for maximum *or* minimum. In calculus, we have two concepts of these *relative* extrema and *absolute* extrema. Let's focus for a second on *absolute* which are stated as:

> A value $y = f(x)$ is an absolute maximum over an interval $[a, b]$ if $y \geq f(x)$ for all x in $[a, b]$. (An absolute minimum has $y \leq f(x)$ instead.)

There are two theorems which help identify extrema here. The first, due to Bolzano, says that any continuous function on a *closed* interval will have an absolute maximum and minimum on that interval. The second, due to Fermat, tells us where to look: these absolute maximums and minimums can only occur at end points or critical points, then evaluate to determine.

Bolzano and Fermat are historic figures. For us, we can plot a function to visually see extrema. The value of Bolzano is the knowledge that yes, plotting isn't a waste of time, as we are *guaranteed* to see what we look for. The value of Fermat is that if you want to get *precise* numeric answers, you have a means: identify the end points and the critical points.

The notes walk you through the task of finding among all rectangles with perimeter 20 the one with maximum area. This is done quickly via:

```
A(b, h) = h * b             # base times height is area
## From P = 2b + 2h -> h = (P -2b)/2
h(b) = (20 - 2b)/2      # Relates perimeter, height, base
A(b) = A(b, h(b))       # express Area in terms of b alone
x = fzero(D(A), [0, 10])    # critical point of A(b) on its domain
```

Here `x` is a critical point. Following Fermat, we would check the value of the function at `x` along with the endpoints, 0 and 10. However, a simple graph also illustrates that any maximum occurs in between these endpoints (with the minimum occurring at both):

```
plot([A, D(A)], 0, 10)      # Notice zero of D(A) corresponds to maximum of A
```

Notice what is done. The original problem had two variables (a base and a height) and a fixed relationship between them (the perimeter is 20). From this one variable can be deduced in terms of another leaving us a continuous function (`A`) with extrema of interest (in this case the maximum).

To solve for the x-value corresponding to the extrema, we used `fzero` with bracketing, as it is guaranteed to converge and it is clear that the interval $[0, 10]$ is a bracket for the derivative function. We could also have identified a good initial guess for the maximum from the graph, say 5, and just called `fzero` with this initial guess, as `fzero(D(A), 5)`.

1.0.2 Questions

For the following questions (which were cribbed from various internet sources), find the most precise answer you can.

- Ye olde post office

A box with a square base is taller than it is wide. In order to send the box through the U.S. mail, the height of the box and the perimeter of the base can sum to no more than 108 inches. What is the maximum volume for such a box?

- How big is that can?

A cylindrical can, open on top, is to hold 355 cubic centimeters of liquid. Find the height and radius that minimizes the amount of material needed to manufacture the can. (These are metric units, so the answer will be in centimeters with 2.54cm=1in.) Do these proportions match those you typically see for a 12 oz can?

- cheap paper cups

A cone-shaped paper drinking cup is to hold 100 cubic centimeters of water (about 4 ozs). Find the height and radius of the cup that will require the least amount of paper. You may be interested to know that the volume of such a cup is given by the volume of a cone formula: $V = (1/3)\pi r^2 h$ and the area of the paper is given by $A = \pi r\sqrt{r^2 + h^2}$.

- inscription

A trapezoid is inscribed in a semicircle of radius $r = 2$ so that one side is along the diameter. Find the maximum possible area for the trapezoid.

Draw a picture of a semicircle and a trapezoid. The trapezoid intersects the circle at $(r, 0)$, $(-r, 0)$, $(r\cos(t), r\sin(t))$ and $(-r\cos(t), r\sin(t))$ where t is some angle in $[0, \pi/2]$. The area of trapezoid is the height times the average of the two bases.

- best size for a cheat sheet

a 3-by-5 card is 15 square inches. For all such rectangular cards with 15 sq. inches what is the optimal size to maximize the printable area if there are 1/4-inch margins all around?

- will you be in the water?

The Statue of Liberty stands 92 meters high, including the pedestal which is 46 meters high. How far from the base should you stand so that your viewing angle, theta, is as large as possible? figure

- getting closer

Let $f(x) = \tan(x)$. Find the point on the graph of $f(x)$ that is closest to the point $(\pi/4, 0)$.