Test 2 covers the material in Chapter 4, chapter 5 and chapter 6 that we discussed. Starting with section 4.4.

A brief, but not complete summary follows:

Probability We learned how random variables can be used to model a data set through a *random sample*. With this we make precise what we mean by a sample from a population, what a parameter is and what a statistic is.

Some language that we learned: The distribution of a random variable, and the mean and standard deviation of a population.

Two key distributions are the binomial distribution and the normal distribution. Each of these is specified completely by two parameters. These are n and p for the binomial and μ and σ for the normal.

The quantile-normal plot can help us decide if a sample comes from a normal population.

Sampling distributions Once a random sample is understood as n independent RVs with the population distribution then a statistic is just a numeric summary of these. The fact that the statistic depends on a random sample means that a statistic is too random. Hence it is described by a distribution and summarized with a mean and standard deviation.

We learned that the sampling disributions of all of the following are approximately normal (sometimes we need n to be large):

$$\bar{x} = \frac{1}{n} \sum x_i, \quad \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Confidence intervals We learned about a $(1 - \alpha)100\%$ confidence intervals for the population mean based on the sample mean when the population standard deviation is assumed:

$$\bar{x} - z^* \sigma / \sqrt{n}$$

The formula is straightforward, but how we interpret them is subtle.

- Significance tests A significance test is another form of inference extracting knowledge about a population from a sample. In this case we have several steps:
 - 1. State a null and alternative hypothesis
 - 2. Under the null a test statistic is chosen with a known sampling distribution
 - 3. Data is collected
 - 4. A p-value is computed
 - 5. An interpretation of the p-value is made.

In addition to producing and intpreting a p-value we discussed issues that can arise (6.3) and the idea of the power (6.4) of a test to detect a certain size effect.

Some sample problems I might ask would be:

1. If X is binomial with n = 5 and p = 1/4 find all of the following:

$$E(X)$$
, $SD(X)$, $P(X=3)$, $P(X \le 3)$

2. If X is binomial with n = 500 and p = 1/4 approximate all of the following:

P(X > 150), P(100 < X < 125)

3. Again, let Z be a standard normal. Find z for each

$$P(Z \le z) = .32, \quad P(Z \ge z) - 0.10$$

4. Let X_1, X_2, \ldots, X_{16} is random sample for a normal population with mean 10 and standard deviation 20. Find the following

$$P(X_1 + X_2 + \dots + X_{16} > 170), \quad P(\bar{x} > 21), \quad P(15 < \bar{x} < 25)$$

- 5. Suppose waist sizes are normally distributed with a mean of 92 cm and standard deviation of 11cm. Let Y denote the average of 15 randomly chosen waists, find
 - (a) $P(Y \ge 100)$.
 - (b) $P(Y \ge y) = 0.80$
- 6. Suppose X_1, X_2, \ldots, X_n is a random sample from a population with mean μ and standard deviation σ . Which of these statements actually makes sense?
 - (a) The sample mean is the population mean.
 - (b) The mean of the sample mean is the population mean.
 - (c) the standard deviation of the sample mean is the population standard deviation.
 - (d) The distribution of the sample mean (for large n) is not the population distribution but the normal distribution.
- 7. A random sample has the following summaries:

> c(m = length(x), xbar = mean(x), s = sd(x))

m xbar s 20.0000000 0.4257475 2.3123897

> qqnorm(x)



If this data set is appropriate for finding a 95% CI for μ based on \bar{x} say why and then find the CI. Otherwise, say why not.

8. A random sample has the following summaries:

$$> x = rexp(200)$$

> c(n = length(x), xbar = mean(x), s = sd(x))

n xbar s 200.0000000 1.0271198 0.9562725

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> qqnorm(x)
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If this data set is appropriate for finding a 95% CI for μ based on \bar{x} say why and then find the CI. Otherwise, say why not.

9. (Stolen from Wikipedia) A machine fills cups with margarine, and is supposed to be adjusted so that the mean content of the cups is close to 250 grams of margarine. Of course it is not possible to fill every cup with exactly 250 grams of margarine. Hence the weight of the filling can be considered to be a random variable X. The distribution of X is assumed here to be a normal distribution with unknown expectation μ and (for the sake of simplicity) known standard deviation $\sigma = 2.5$ grams. To check if the machine is adequately calibrated, a sample of n = 25 cups of margarine is chosen at random and the cups weighed. The weights of margarine are X_1, \ldots, X_{25} , a random sample from X. The average of these 25 values is 250.2 grams.

Find a 90% CI for μ .

10. Wikipedia's entry has:

We cannot say: "with probability $(1 - \alpha)$ the parameter μ lies in the confidence interval." We only know that by repetition in $100(1 - \alpha)\%$ of the cases μ will be in the calculated interval. In 100α % of the cases however it doesn't. And unfortunately we don't know in which of the cases this happens. That's why we say: "with confidence level $100(1 - \alpha)$ % μ lies in the confidence interval."

Did they get it right? If not, can you correct them where you think it is wrong?

- 11. A student is interested in finding a 95% CI for an unknown mean time to find a parking spot on campuse. They know that $\sigma = 5$ and assume a normal distribution. How large a sample is needed to produce a CI with a margin of error of 1 minute?
- 12. (Wikipedia again). We have this statement:

In statistical hypothesis testing, the p-value is the probability of obtaining a test statistic at least as extreme as the one that was actually observed, assuming that the null hypothesis is true. The fact that p-values are based on this assumption is crucial to their correct interpretation.

The lower the p-value, the less likely the result, assuming the null hypothesis, so the more "significant" the result, in the sense of statistical significance. One often rejects a null hypothesis if the p-value is less than 0.05 or 0.01, corresponding to a 5% chance or 1% of an outcome that extreme, given the null hypothesis.

We've said in class "The *difference* is statistically significant" here they say the "result." Are we talking about the same thing?

- 13. Suppose a test has been given to all high school students in a certain state. The mean test score for the entire state is 70, with standard deviation equal to 10. Members of the school board suspect that female students have a higher mean score on the test than male students, because the mean score from a random sample of 64 female students is equal to 73. Does this provide strong evidence that the overall mean for female students is higher? State the hypothesis and show how you compute a p-value. Assume the population standard deviation for females is the same as for males.
- 14. A student wants to know if parking takes longer this semester than last, as several changes have occured: more students, different traffic patterns, more students taking the shuttle, Last semester, extensive evidence found that the time to park was summarized by $\mu = 10$, $\sigma = 5$ with a basically normal distribution. In a sample of 16, a sample average of 12 minutes was found. Is this difference statistically significant at the 0.05 level? Write out H_0 , H_a , find the *p*-value, ...
- 15. (Power) The last problem is calculated assuming $\mu = 10$. The power of that test to detect an effect of size 1 is given by repeating the *p*-value calculation with an assumption that $\mu = 11$. Can you find the power? Compare your value to the following from Wikipedia:

Although there are no formal standards for power, most researchers assess the power of their tests using 0.80 as a standard for adequacy.