[Disclaimer: this study sheet is not intended to contain all the possible questions that may appear on the test. It is only intended to help you get started on studying for the exam.]

The second test in MTH 231 will cover the following lessons from the syllabus: 11-27. That is, sections 1.5, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 3.1, 3.2, 3.3.

The test will be for a full period and will have 100 possible points in roughly 20 questions.

What to study? Here is an overview of the material with a few sample questions indicated from the book, or written by me.

Infinite limits This didn't get asked last time round. There are two types of infinite limits. Both correspond to asymptotes of the graph of a function. A horizontal asymptote descibes what happens to a function as x heads to plus or minus infinity. The horizontal asymptote is a line with height given by the limit at plus infinity (or minus sometimes).

A vertical asymptote is when a function "blows up." That is heads to plus or minus infinity somewhere on the x axis. Think $1/x$ at 0. These are related to limits of inifinity.

Questions

1. Why the difference in limit at infinity and limit of inifinity?
2. Write down a function with a limit at infinity of 0
3. Write down a function with a right limit at infinity of 1
4. Find
$$\lim_{x\to\infty} \frac{x^2+2}{x^2-3}, \lim_{x\to-\infty} \frac{x+2}{2x-3}, \lim_{x\to 1+} \frac{x^2+x-2}{(x-1)^2} \lim_{x\to 1+} \frac{\sin(x)}{x-1}$$
5. Sketch the graph of $(x^2_2x+1)/(x^3-3x^2)$ as x goes to infinity (it has an asymptote).
6. Problems in book 1.5: 13, 43, 59 say.

Basic definition of the derivative The key defn of a derivative in terms of the limit of a secant line gives us insight into what the derivative is – it tells us the slope of the tangent line, when it exists.

The defn involves a limit. You should expect to do atleast one of these problems using the definition.

The analogy we used in class was average speed versus speedometer speed (instantatneous). The average speed is found as a slope of a secant line for the time versus position graph, the speed its rate of change.

1. We have 3 basic concepts: At a point, a function can have a limit, be continuous, or have a derivative. There is a logical implication in that if a function has a derivative at $x = c$ then it is continuous and hence has a limit at $x = c$. The "reverse" is decidedly not so. Give an example of a function which has a limit at $x = 0$ but is not continuous there. Give an example of a function continuous at $x = 0$ which does not have a derivative at $x = 0$.
2. Use the defn. of a derivative to compute $f'(3)$ when $f(x) = 14x$.

3. The many notations for the derivative are listed on page 99. Which of these is not:
$$f'(x), \frac{dy}{dx}, y', \frac{d}{dx}[f(x)], D_x[y], f(dx)?$$
4. On page 97 and 99 are two definitions of a derivative. Explain how they are the same (one uses delta x, the other $x - c$.
5. A convex function looks like a "U" (for example x^2). Using the definition of a derivative explain why the slope of the secant line from x to $x+1$ is always more than the derivative of the function at x. (Draw a picture.)
6. The derivative of 2^x is one we can't take from our basic rules. Use your calculator to make a table of values to estimate the value of the derivative at $c = 0$. (The answer is not an integer, so try to figure out the first decimal point.)
7. Why isn't the following true $[2^x]' = x2^{x-1}$?

Rules for taking derivatives The basic derivative formula can be cumbersome to use. However, there are many rules for taking derivatives that making it a snap to do – if you are in practice. We have rules to differentiate constants, sums and differences, products and divisors and composition.

These questions may make up as much as 35 percent of the exam – so make sure you can do all of these – and more.

Find $f'(x)$ when: (don't simplify)

1. $f(x) = \pi^2$
2. $f(x) = x^2$
3. $f(x) = (x + 3x^3)^{-1/2}$
4. $f(x) = (x^2 - 3x)\sqrt{x^2 - 2}$
5. $f(x) = (x^2 - 2)/(x^2 + 2)$
6. $f(x) = \sin(x^2 - 2x + 3)$
7. $f(x) = \tan^2(\sin(\pi x))$
8. $f(x) = \sin(\sin(\sin(x)))$

Find $f'(1)$ when:

1. $f(x) = (x + 1)(x - 2)$
2. $f(x) = (x + 1)/(x - 2)$
3. $f(x) = ((x + 1)^2 - 1)^3$

Find an equation for the tangent line to the graph $f(x) = x^2$ when $x = 1$. From your equation, find the point where this line intersects the x axis.

Implicit differentiation and related rates The chain rule allows one to extend the concept of the derivative returning the slope of a tangent line to the graph of a function to the graph of an equation. Then dy/dx refers to this value for particular solutions (x, y).

Related rates uses the chain rule to turn an equation involving specific quantities – say (volume and radius) to the rate of change in timre for these quantities.

For both the basic trick is if you have an equation, then one can differentiate both sides to get a new equation. For implicit differentiation, you take $/dx$ of both sides, whereas for rates you take d/dt.

1. In an implicit differentiation problem the derivative of x is 1, what about in related rates, when you take d by dt? Extend this, what happens to a term like x^3 for both implicit differentiation and related rates?
2. For the equation $x^2 + 3y^2 = 4$, find the slope of the tangent line at $(1, 1)$.
3. A snowball melts at a rate proportional to its surface area (for easy, we set this proportion to 1), where

$$V = \frac{4\pi}{3} r^3, \text{ and} \frac{dV}{dt} = S = 4\pi r^2.$$

Find dr/dt in terms of dV/dt and r, but not in terms of S. (Algebra). Is it constant?

Extrema on an interval This section introduced the word extrema for maxima and minima. Of course, these needed definitions. We had

- maxima on an interval
- relative maxima
- absolute maxima

Then we had two main theorems: the extreme value theorem said that a continuous function on a closed interval $[a, b]$ had a maximum value; another said a relative extrema only occurs at a critical number.

1. Can you draw a function on $[0, 1]$ with an absolute maximum at an endpoint? How about a function with an absolute max not at an endpoint? How about a function with a relative max at $1/3$, but this is not an absolute max?
2. On the interval $(0, 1)$ – open! – draw a function that is continuous, but has no maximum.
3. Find all critical numbers of $f(x) = (x - 2)(x - 3)$.
4. Find all critical numbers of $f(x) = \sin(x) - x^2/2$.
5. Find all critical numbers of $f(x) = |x - 3|$.

6. Use the extreme value theorem to find all extrema for $f(x) = x^2 - 4x + 10$ on the interval $[0, 5]$.
7. Find the maximum and minimum values of $f(x) = x^4 3x^3 1$ on $[2, 2]$.

Rolle's Theorem, Mean value theorem The mean value theorem says that for a continuous function on a closed interval that is differentiable that any secant line (average velocity) is matched by a parallel tangent line (instant velocity).

Be prepared to identify the point where the tangent line has the slope of a secant line.

First derivative test The first derivative test comes from the relationship between increasing functions and their derivatives. Namely

1. If $f(x)$ is increasing on an interval, then $f'(x) \geq 0$ on the interval
2. If $f'(x) > 0$ on an interval then $f(x)$ is increasing on the interval
3. And, if $f'(x) = 0$ on an interval, then $f(x)$ is constant on the interval.

The symmetry is broken, as an increasing function can pause so it is possible $f'(x) = 0$ at a point – but not an interval.

The first derivative test then characterizes critical numbers – where $f'(x)$ is 0 or DNE. For continuous $f'(x)$, this where the derivative can possibly change signs. If the signature is

+	cp	-	a relative max
-	cp	+	a relative min
+	cp	+	not a relative max or min
-	cp	-	not a relative max or min

The key to using this test is to find out exactly when the first derivative is 0 or DNE (critical points) and then on what intervals is it + or -.

1. $f(x) = 3x^4 - 4x^3 - 12x^2 + 3$ Use the first derivative test to characterize all critical values as relative max or mins.
2. If $f(x) = x^4 8x^2$, determine all local extrema for the function.
3. Let $f(x) = \sin x + \cos x$ Find all local extrema on the interval $[0, 2\pi]$.

I'm writing this waiting for some kids to ring my doorbell so I can give them candy. Keep this in mind and try to answer:

Compute the indicated derivative based on the following definitions of f, g and h. Write your answer very carefully paying special attention to capitalization and parentheses.

$$\begin{array}{ll} f(x) = \mathbf{H}(x) & f'(x) = \mathbf{h}(x) \\ g(x) = \mathbf{ve}(x) & g'(x) = \mathbf{ppy} \text{ (a constant)} \\ h(x) = x & h'(x) = \mathbf{lloween} \text{ (a constant)} \end{array}$$

1. Compute $(fg + f \circ h)'(A)$.