Test 2 in MTH 339 covers material from the end of chapter 4 through the material in chapter 7. The test will have proofs. As before, you will be allowed to bring in a 4 by 6 card with notes. If you still have your previous one, you may bring that as well.

What follows are some sample problems for you to chew over. They are broken down by calculations, then proofs. There are many more things here than I could possibly ask on a test. Make sure you can do the first group, as those skills will definitely be on the test.

1. Let $|G| = 42$ be cyclic and generated by a. Describe all the subgroups of G.
2. Let $|G| = 100!$ be cyclic and generated by a. (That's a **big** group) How many elements are there of order 24?
3. For the clock group Z_{12} describe completely all proper subgroups. Is every element in a proper subgroup?
4. For the permuation

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| 1 2 3 4 5 6 7 8 9 |
| 5 4 6 2 7 1 8 9 3 |
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   (a) Write this in cycle notation using disjoint cycles
   (b) Write this as a product of 2-cycles
   (c) Find the inverse of this permutation
   (d) What is the order or this permutation

5. The alternating group A_4 has 12 elements, 8 of which have order 2. List them.
6. The symmetry group of a cube can be viewed as a subgroup of S_6 by identifying a symmetry as a permutation of the 6 faces. Write down 3 permutations of order 4 in cycle notation that generate this group.
7. Let R_* be the group of real numbers without 0 under multiplication. Which of these, if any, is an automorphism?

   (a) $f(x) = x^3$
   (b) $f(x) = \sqrt{x}$
   (c) $f(x) = exp(x)$?

8. In A_4 is the element (123). Describe the permutation $T_{(123)}$ by how it acts on the other elements of order 2 in A_4. (There are 7 others).
9. In Z_9 describe the cosets of $< 3 >$. (Why did I not say left or right?)

10. Use Fermat's little theorem to compute $5^{22} mod 7$.

11. List the possible subgroups sizes for a group of order 42.

12. Let $G = D_5$. Without working too hard can the following be a subgroup? $H = \{e, r, r^3, r^4\}$?

13. Let G be a subgroup of S_4 containing $\{(13)(24), (14)(23)\}$.
    (a) How many elements does G have?
    (b) Describe $stab_G(1)$
    (c) Describe $orb_G(1)$.

Now for proofs

1. Suppose $|a| = n$. Find a necessary and sufficient condition of r and s such that $< x^r > \subset < x^s >$.

2. If α is an even permutation, what is α^{-1}.

3. Let T be a function from S_n into Z_2 defined by $T(\alpha) = 0$ if α is even, and $T(\alpha) = 1$ if odd. Show that T preserves the group property ($T(\alpha\beta) = T(\alpha) + T(\beta)$).

4. Show that a permutation with odd order must be an even permutation.

5. Let G be a group. Show that the mapping $\alpha(g) = g^{-1}$ is a automorphism if and only if G is Abelian. Describe this mapping geometrically on the group of non-zero rational numbers under multiplication.

6. Find $AUT(Z)$. (This is a cyclic group.)

7. Suppose both g and h produce the same inner automorphism of G. (That is $\phi_g(a) = gag^{-1} = \phi_h(a) = hah^{-1}$ for all a in G. Show $h^{-1}g$ is in the center of G.

8. Let G be a group of order 49. Show that G is cyclic, or $a^7 = e$ for all a in G.

9. If $|G| = 2 \times 5$ and G has only one subgroup of order 2 and order 5 prove that G is cyclic.

10. If $|G| = pq$, primes, show any proper subgroup is cyclic.

11. Can a group have more subgroups than elements?

12. Suppose $G = \{e, a_1, a_2, \ldots, a_{2k}\}$. (So $|G|$ is odd.) Show that $ea_1a_2 \cdots a_{2k} = e$

13. Suppose G has $|G| = p^n$. Show that the center, $Z(g)$ can not have index p. (Hint, this is harder. It takes after the proof of theorem 7.2).