Name: MTH339 Final exam review

Some sample questions for you to practice on. These only cover the material since the last test. Good luck. If desired solutions can be posted online, as before.
These use the Fundamental Theorem of Abelian Groups:

1. How many elements of order 2 are there in: Z_{16}, $Z_8 \oplus Z_2$, $Z_4 \oplus Z_4$
2. Prove that any Abelian group of order 45 has an element of order 15. The same is true, of course, for cyclic groups. Do you expect it is true for every group?
3. Find all Abelian groups of orer 360.
4. The set $\{1,9,16,22,29,53,74,79,81\}$ is a subgroup of $U(91)$. Determine it's isomorphism class.
5. Which Abelian groups of order n are *necessarily* cyclic?
6. Without using Lagrange's Theorem, but using the Fundamental Theorem of Abelian Groups, show that any Abelian group of odd order cannot hava an element of even order.

This is a group homomorphism problem. Although I can't test you on the first Isomorphism theorem, I can ask questions such as

1. Let $\phi : D_n \to Z_2$ be defined by $\phi(r^i f^j) = j \mod 2$. Show that ϕ is a group homomorphism. Find its kernel.
2. If ϕ is a homomorphism from G to $\bar{G}$ and $K \leq G$. Show $\phi(K)$ is a subgroup of $\bar{G}$.
3. Explain why $Z_n = Z/<n>$, without using the First isomorphism theorem.
4. If k divides n find an isomorphism from Z_n onto Z_k with kernel $<k>$.

These are problems about Normal groups.

1. If $H \subset G$ and $G : H = 2$ show H Is normal.
2. What is the order of the element $14+<8>$ in the factor group $Z_{24}/<8>$?
3. Suppose H, K are normal subgroups of G with empty intersection that are both Abelian. Show $HK = H \times K$ is Abelian.
4. Prove that D_4 can not be expressed as an internal direct product of two groups. (Hence if K is a normal subgroup of D_4 and $D_4/K = H$ there is no way of expressing D_4 as $H \times K$.) You can do this by looking at all the subgroups: $<r>, <r^2>, <f>, <rf>$ $, <r^2 f>, <f^3 f>$ and using a previous worksheet problem.

5. A *simple* group is one with no non-trivial normal subgroups. A_n is simple if $n \geq 5$. What is $Inn(A_n)$ isomorphic to?
6. In the Rubik's cube group (the group of motions of the cube) the subgroup of actions that stabilize the 8 corners is a normal subgroup. Why? (Show ghg^-1 is in this group if h is.) The quotient group is a 2 by 2 Rubik's cube.