Test 2 covers the following topics:

The counting formulas We haven't tested on ${}_nP_r$ and ${}_nC_r$. They will be on the test

Discrete distributions and random variables We learned that we can describe a random variable by its distribution. When the random variable is discrete, then the distribution is described by $P(k)$ or $P(X=k)$ for each possible value of k that X can be.

We can summarize a probability distribution by the mean μ and standard deviation σ whose formulas are given by

$$\mu = \sum kp(k) = \sum_k kP(X=k), \quad \sigma^2 = \sum_k (k-\mu)^2 p(k) = \sum_k (k-\mu)^2 P(X=k), \quad \sigma = \sqrt{\sigma^2}.$$

You should expect to compute a value of μ and σ.

The binomial distribution This describes the number of successes in n independent trials, where each trial has success probability p. This is our main example of discrete distribution. For this distribution, we know each of $P(X=k)$, μ and σ by formulas:

$$P(X=k) =_n C_k p^k (1-p)^k, \quad \mu = np, \quad \sigma = \sqrt{np(1-p)}.$$

Continuous distributions A continuous distribution describes continuous random variables. The probabilities are specified using the *area* under some curve called the density. We do not have formulas to compute the mean and standard deviation, although these concepts exist to describe the center and spread of a continuous distribution.

The normal distribution The bell shaped curve describes the family of normal distributions, our primary example – but not only – of a continuous distribution. A special case is the standard normal for which the "table" in the book is provided to compute areas.

For a general normal, it is characterized by two values: the mean μ and standard deviation σ. The general normal has its probabilities also answered by areas under the density. For this, the standard normal table can be used by considering z scores:

$$z = \frac{x-\mu}{\sigma}, \quad x = \mu + z\sigma.$$

Remember to get partial credit – draw a picture. A standard normal distribution looks like the figure.

The normal approximation to the binomial As a first application for the normal distribution, we saw that if X is a binomial random variable with parameters n and p then probabilities for X can be *approximated* by probabilities for a normal random variable

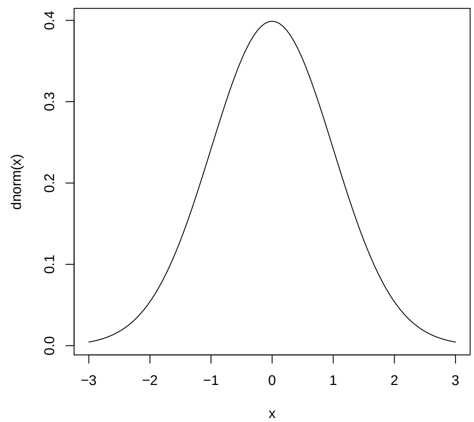

Figure 1: Standard normal distribution

with parameters $\mu = np$ and $\sigma = \sqrt{np(1-p)}$. To be concrete if X is a binomial, and Z a standard normal then

$$P(X \leq a) \approx P(Z \leq \frac{a-\mu}{\sigma}) \quad \text{and} \quad P(X \geq a) \approx P(Z \geq \frac{a-\mu}{\sigma})$$

The book includes a $+1/2$ and $-1/2$. This makes the approximation more accurate, but to simplify matters we ignore it. (In practice, the for large n it is negligible, and if needed exact answers can be found using a computer.)

Some sample problems I might ask would be:

1. The age distribution for incoming CSI students is roughly

Age	17	18	19	20	21	22
Probability	.1	.4	.3	.1	.05	.05

   Find the average age of a randomly chosen incoming CSI student.

2. A random number is chosen from 1,2,3,4,5 with each being equally likely. Let X be the value. What is the standard deviation of X? (The mean is 3!).

3. Which of these random variables is described by the binomial distribution? If possible identify n and p when it is binomial.

   (a) A coin is tossed 10 times, let X be the number of heads.

   (b) A researcher surveys the first X students until they get 100 who are going to vote in the upcoming election.

(c) A researcher surveys 100 students, let X be the number who are going to vote in the upcoming election.

4. If X is binomial with $n = 5$ and $p = 1/4$ find all of the following:

$$E(X), \quad \mathsf{SD}(X), \quad P(X = 3), \quad P(X \leq 1)$$

5. Suppose X is binomial with $n = 4$ and $p = 1/2$ and Y is binomial with $n = 6$ and $p = 1/3$. Which is more likely $P(X = 2)$or $P(Y = 2)$?

6. To see how effective text-messaging is for contacted students, 100 text messages were sent to 100 randomly chosen students. If the probability of being read is $p = .75$ compute the expected number read. Find the z score for 80 being read.

7. Let Z be a standard normal Find the following:

$$P(Z < 1), \quad P(Z \leq 2.3), \quad P(Z \geq 1.23), \quad P(-1 \leq Z \leq 1/2)$$

8. Again, let Z be a standard normal. Find z for each

$$P(Z \leq z) = .32, \quad P(Z \geq z) = 0.10$$

9. Let Y be a normal random variable with mean 10 and standard deviation 20. Find

$$P(Y > 10), \quad P(Y > 20), \quad P(Y > 31), \quad P(15 < Y < 25)$$

10. Suppose waist sizes are normally distributed with a mean of 92 cm and standard deviation of 11cm. Let Y denote a randomly chosen waist.

    (a) Compute $P(Y \geq 100)$.

    (b) Find y so that $P(Y \geq y) = 0.80$

11. It is known that 25% of CSI students are 20 or younger. An event is held and 100 students show up. What is the probability that there are 26 or more students 20 or younger *if* the binomial distribution describes the number that do show up? Use the normal approximation to answer this with a number.

12. A survey of 1000 college students is taken. In past elections 35% of college students have voted. If this same percentage applies to this survey, find the probability that 325 or fewer will vote in the upcoming election. Assume that the number who will vote is binomial and use the normal approximation to give a numeric answer.