Homework problems: I've done a few of the ones that caused troubles.

Let X, Y be iid normal with mean μ and variance σ^2. Show $X+Y$ is normal.

First, we know $E(X+Y) = 2\mu$ and $var(X+Y) = 2\sigma^2$. By transformations, it is enough to consider the case $\mu = 0$, $\sigma = -1$.

$$\begin{aligned} P(X+Y \leq a) &= \iint_{x+y\leq a} f(x)f(y)dxdy \\ &= \int_{-\infty}^{a}\int_{-\infty}^{a-x} f(x)f(y)dxdy \end{aligned}$$

Let $u(1,1)+v(1,-1) = (x,y)$ be a change of variables with jacobian 2. This is chosen so the new limits of integration are rectangular. Then the above transforms into

$$\begin{aligned} P(X+Y \leq a) &= \int_{-\infty}^{a/2}\int_{-\infty}^{\infty} f(u-v)f(u+v)2dvdu \\ &= \int_{-\infty}^{a/2}\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}}e^{(1/2)(\sqrt{2}u)^2}\frac{1}{\sqrt{2\pi}}e^{(1/2)(\sqrt{2}v)^2}2dvdu. \end{aligned}$$

This separates into two integrals. For the v one, change variables with $x = \sqrt{2}v$, and use $\int f(x)dx = 1$ to get $1/\sqrt{2}$.

For the u one, change variables with $x = 2u$. Then we get

$$P(X+Y \leq a) = 2\frac{1}{\sqrt{2}}\int_{-\infty}^{a}\frac{1}{\sqrt{2\pi 2}}e^{-\frac{1}{2\cdot 2}x^2}dx$$

But this is $F(a)$ for a normal random variable with mean 0 and variance 2.

7.7 – no issue here except the data set is a chore.

7.15 – ditto

7.17 – Why is this not the median of the total population? Not because there isn't enough data, but because the median value of medians need not be the median for the entire set as a whole. A simple example is the data sets 1,1,1 and 4,4,4,4. The median of first is 1, second 4 so median of both would be (1+4)/2 which is not 4 – the median of all 7 values. For means we can combine means by weighted sums. With medians it isn't so simple.

7.19 – same comment. As well, let's agree that if there are 50 values in a data set, you randomly sample 10 to do things with, unless you can get the data in digital format.

7.25 – no issues

7.27 – s.d. is easier to compute using computer, of course. Use it if you want (sd()).

7.32 – We did this in class: differentiate $f(x)$ and solve for $f;(x) = 0$.

7.37 – last correlation is 0 by symmetry. (Draw lines through $\bar{x}$ and $\bar{y}$ and see how points go through these lines.

7.38 – Using cor():

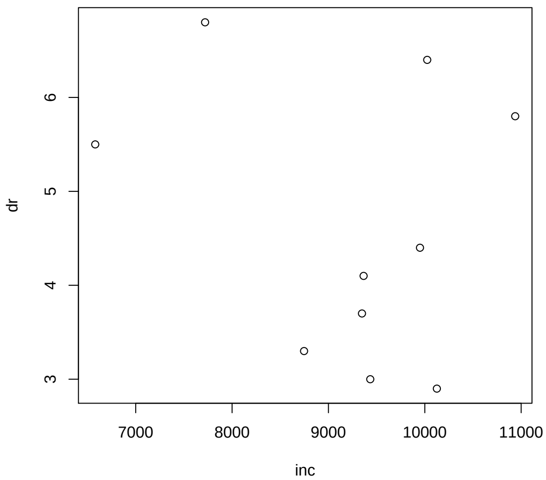
```
> inc = c(7720, 10938, 10125, 9348, 9950, 9365, 6580, 10025, 9434,
+     8747)
> dr = c(6.8, 5.8, 2.9, 3.7, 4.4, 4.1, 5.5, 6.4, 3, 3.3)
> cor(inc, dr)

[1] -0.2296

> torm = c(1, 7)
> cor(inc[-torm], dr[-torm])

[1] 0.5976

> plot(inc, dr)
```



(using negative indexing to exclude first and seventh numbers.)

R.3 – Assume that $\sum x_i = 0$. (Otherwise, we can look at $y_i = x_i - \bar{x}$.) Then

$$\begin{aligned}
\sum_i \sum_j (x_i - x_j)^2 &= \sum_i \sum_j x_i^2 - 2x_i x_j + x_j^2 \\
&= \sum_i n x_i^2 + \sum_j n x_j^2 - 2 \sum_i x_i \sum_j x_j \\
&= 2 \sum_i n x_i^2 - 2 * 0 \\
&= 2n(n-1)S^2
\end{aligned}$$

Now fuss with the factors in front to match the book.