7-R11 The derivative (in a) of $|X_i - a|$ is a heaviside function (-1 if less than X_i, 1 if more and undefined if equal to X_i. So for a less than $X_{(1)}$ the derivative is $-n$, for a more than $X_{(n)}$ the derivative is $_n$, and whenever a crosses over an $X_{(i)}$ the derivative jumps up by 2.

Now, 1st-year calculus applied here says that a minimum can only occur at an endpoint, or a critical point. Further more, the first derivative test says that a critical point is a relative extrema only if the derivative changes sign.

Now look at cases. Suppose the n points are distinct and $n = 2k+1$, Then $X_{(k+1)}$ is the median, a critical point, and such f' to the left of this point is equal to $-n+2*k = -2k-1+2k = -1$ and to the right is $-n+2(k+1) = 1$. So the derivative changes sign at the only possible critical point so the value is a relative minimum, hence a global minimum.

If n is even, say $2k$, then between X_k and X_{k+1} the derivative is equal to $-n+2k = 0$. So any value in this interval is a critical point. In fact the function $f(a)$ is flat there. In particular the median value will be a minimum value.

8.1 Key is a random sample has joint density given by

$$f(x_1, x_2, \ldots, x_n|\theta) = \prod f(x_i|\theta)$$

In particular for a we have

$$f(x_1, x_2, \ldots, x_n|\theta) = \prod_{i=1}^{n}(1/2), -1 \leq x_i \leq 1 = (1/2)^n, -1 \leq x_i \leq 1.$$

8.3 The likelihood function is proportional to joint density so it can always be taken to be

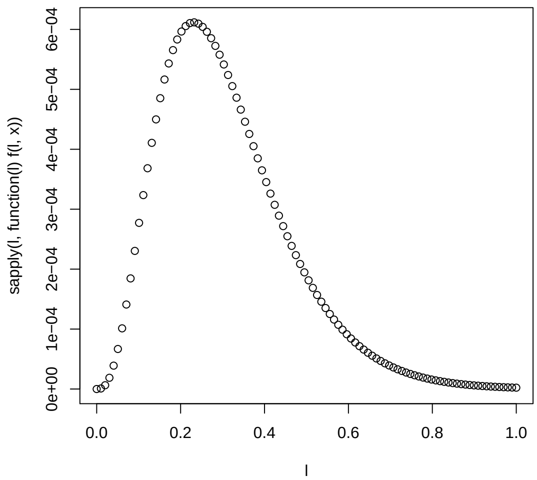
$$L(\theta) = f(x_1, x_2, \ldots, x_n|\theta) = \prod f(x_i|\theta).$$

For the exponential this is

$$\lambda e^{-2\lambda} \cdot \lambda e^{-4\lambda} \cdot \lambda e^{-7\lambda} = \lambda^3 e^{-(2+4+7)\lambda}$$

To visualize we can use `dexp()`. Here's one way using functions.

```
> f = function(l, x) {
+     prod(dexp(x, rate = l))
+ }
> x = c(2, 4, 7)
> l = seq(0, 1, length = 100)
> plot(l, sapply(l, function(l) f(l, x)))
```



8.6 For a sample of size 10, the likelihood function is the product of 10 densities. For instance the gamma becomes

$$\begin{aligned} L(\alpha,\lambda) &== \prod (\lambda^a/\Gamma(\alpha)) x_i^{\alpha-1} e^{-\lambda x_i} \\ &= (\lambda^a/\Gamma(\alpha))^n e^{-\lambda \sum x_i} (\prod x_i)^{\alpha-1}. \end{aligned}$$

8.9 We need to show that the density can be written in the form of the formula in example 8.3e. For instance (c) has

$$f(k,\lambda) = \frac{\lambda^k}{k!} e^{-\lambda k}.$$

So $B = \lambda^k$, $h(k) = 1/k!$, $Q = -\lambda$, and $R(k) = k$.

So $\sum R(k_i)$ is sufficient.

8.11 Key is the statistic is a pair $(\sum x_i, \sum y_i)$

8.13 Follow example 8.3f, but here we have $L(a,b) = \frac{1}{(b-a)^n}$ for $a \le x_{(1)}, b \ge x_{(n)}$. So the pair (min,max) is sufficient.

8.19 class notes

8.22 skip this one

8.24 We have the distribution of $X_{(5)}$ for this is given by

$$\begin{aligned} P(x_{(5)} < 1.2) &= P(x_1, x_2, x_3, x_4, x_5 < 1.2) \\ &= \prod P(x_1 < 1.2) \\ &= \Phi(1.2)^5 \end{aligned}$$

Where $\boldsymbol{\Phi}$ is c.d.f. for standard normal:

```
> pnorm(1.2)^5

[1] 0.5427
```