This test will cover the following topics:

1. 11.4: related rates
2. 11.5: Application to business. This covered the topic of elasticity of demant and its definition: $\nu = -(p/q)(dq/dp)$.
3. 12.1: the indefinite integral $\int f(x)dx$
4. 12.2: the power rule
5. 12.3: integrating exponential and logarithmic functions
6. 12.4: application to $\bar{MR} = \bar{MC}$ (skip the part on marginal propensity to consume/save)
7. 12.2: the definite integral and fundamental theorem of calculus

Careful, there will be word problems due to the applications in 11.4, 5, and 12.4.

The main new concept was the idea of an **anti derivative**. That is some function $F(x)$ whose derivative is $f(x)$. That is $F'(x) = f(x)$.

We have notation for this

$$\int f(x)dx$$

stands for any function which is an antiderivative of $f(x)$. This is why it has a $+C$ in the answers.

In contrast the notation

$$\int_a^b f(x)dx$$

is the definite integral, and evaluates to a number which is the area under $f(x)$ over the interval $[a,b]$.

The fundamental theorem of calculus tells us how to evaluate this area

$$\int_a^b f(x)dx = F(b) - F(a) = F\|_a^b$$

That is, any antiderivative can be used to find area.

The key to using the above is knowing antiderivatives. In class we learned that

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1,$$

$$\int \frac{1}{x} dx = \ln(x) + C$$

$$\int e^x dx = e^x + C$$

$$\int cf(x)dx = c\int f(x)dx$$

$$\int (f(x)+g(x))dx = \int f(x)dx + \int g(x)dx$$

By doing substitutions, we can reverse the chain rule. The basic idea is to let u be some part of the integral, and then look at the relation given by the differentials. (If $u = f(x)$ then $du = f'(x)dx$.)

Here are some problems to work on.

First some indefinite integrals.

1. $\int x^2 dx$
2. $\int \sqrt{p} dp$
3. $\int u^{100} du$
4. $\int e^t dt$
5. $\int (16x^2 - 16x - 16/x) dx$
6. $\int (x^2 - x + 1 - 1/x + 1/x^2) dx$
7. $\int p(1-p) dp$

These require substitutions

1. $\int e^{-5x} dx$
2. $\int xe^{-5x^2} dx$
3. $\int 1/(5-x) dx$
4. $\int (x^2 - x)/(x^3 - (3/2)x^2) dx$
5. $\int (x^2 - x)(x^3 - (3/2)x^2)^3 dx$
6. $\int (x^2 - x)/(x^3 - (3/2)x^2)^3 dx$
7. $\int \sqrt{u^2 - 1} u du$
8. $\int \frac{1}{x \ln(x)} dx$

These are answered with a number

1. $\int_0^{10} x dx$
2. $\int_0^1 x^{100} dx$
3. $\int_{-1}^1 x^3 dx$
4. $\int_0^3 e^x dx$
5. $\int_1^e 1/x dx$

These might need a substitution first.

1. $\int_2^3 x(x^2 - 2)^{1/2} dx$

2. $\int_1^3 x/(x^2+2)dx$
3. $\int_1^4 (16x^2-16x+15)dx$

These integrals find the area of some geometric figure. Draw the figure. (You don't need to find the integral)

1. $\int_0^1 dx$
2. $\int_0^5 x dx$
3. $\int_{-1}^1 \sqrt{1-x^2}dx$

Word problems.

1. A pebble drops in the water and radial circles emanate. If the area of the circle is $A = \pi r^2$ and the radius is changing at a constant rate of 2, what is the rate of change of A when $r = 1$?
2. A cells volume is related to its radius by
$$V = \frac{4}{3}\pi r^3$$
If dV/dt is a constant of 1, what is dr/dt when $r = 1$?
3. A demand function $pq = 27$, where p is in dollars, q in number of units.
   (a) Find the elasticity of demand at $(9,3)$.
   (b) How will a price increase affect total revenue, when $R = pq$?
4. The marginal revenue $\bar{MR}$ is $R'(x) = 23 + 20x$. Find $R(x)$ under the assumption that revenue is 0 if no units are sold.
5. The marginal cost $\bar{MC}$ is $C'(x) = 2 + 3/x^2$ for $x \geq 1$. We know $C(10) = 10$. Find $C(x)$.
6. Marginal revenues and cost are given by
$$\begin{aligned} \bar{MC} &= 6x + 60, & C(10) &= 1000 \\ \bar{MR} &= 180 - 2x & R(0) &= 0. \end{aligned}$$
   (a) The optimal profit is found where $P'(x) = 0$. Find all points where $P'(x) = 0$ by solving $\bar{MC} = \bar{MR}$.
   (b) Solve for $P(x)$ using the conditions $R(0) = 0$, and $C(10) = 1000$.
   (c) Is $P(x)$ positive at the optimal point of profit?