1 $\int x^2 dx = \frac{x^3}{3} + c$

2 $\int \sqrt{p}\, dp = \int p^{1/2} dp = \frac{p^{3/2}}{3/2} + c = \frac{2}{3} p^{3/2} + c$

3. $\int u^{100} du = \frac{u^{101}}{101} + c$

4. $\int e^t dt = e^t + c$

5. $\int (16x^2 - 16x - \frac{16}{x}) dx = 16\frac{x^3}{3} - \frac{16x^2}{2} - 16 \ln x + c$

6. $\int (x^2 - x + 1 - \frac{1}{x} + \frac{1}{x^2}) dx = \frac{x^3}{3} - \frac{x^2}{2} + x - \ln x + (\frac{x^{-1}}{-1}) + c$

7. $\int p(1-p) dp = \int (p - p^2) dp = \frac{p^2}{2} - \frac{p^3}{3} + c$

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1 $\int e^{-5x} dx = \int e^u \frac{du}{-5} = e^u \cdot \frac{1}{-5} = -\frac{1}{5} e^{-5x} + c$

let $u = -5x$

$du = -5\, dx$

2. $\int x e^{-5x^2} dx = \int e^u \frac{du}{-10} = -\frac{1}{10} e^u + c = -\frac{1}{10} e^{-5x^2} + c$

let $u = -5x^2$

$du = -5 \cdot 2x\, dx$

3 $\int \frac{1}{5-x} dx = -\int \frac{1}{u} du = -\ln u = -\ln(5-x) + c$

let $u = 5-x$

$du = -dx$

4 $\int \frac{x^2-x}{x^3-\frac{3}{2}x^2}\,dx = \int \frac{\frac{du}{3}}{u} = \frac{1}{3}\int \frac{1}{u}du$

$= \frac{1}{3}\ln u$

$= \frac{1}{3}\ln\left[x^3-\frac{3}{2}x^2\right] + C$

let $u = x^3-\frac{3}{2}x^2$

$du = (3x^2-3x)dx$

$= 3(x^2-x)dx$

5. $\int \frac{(x^2-x)}{\cancel{x^3-\frac{3}{2}x^2}}(x^3-\frac{3}{2}x^2)^3 = \int u^3\frac{du}{3} = \frac{1}{3}\frac{u^4}{4} = \frac{1}{12}\left[x^3-\frac{3}{2}x^2\right] + C$

let $u = x^3-\frac{3}{2}x^2$

$\frac{du}{3} = (x^2-x)dx$

6 $\int \frac{(x^2-x)}{(x^3-\frac{3}{2}x^2)^3} = \int \frac{du/3}{u^3} = \frac{1}{3}\frac{u^{-2}}{-2} = -\frac{1}{6}\left[x^3-\frac{3}{2}x^2\right]^{-2}$

$+ C$

let $u = x^3-\frac{3}{2}x^2$

$\frac{du}{3} = (x^2-x)dx$

7 $\int u\sqrt{u^2-1}\,du = \int \sqrt{v}\,\frac{dv}{2} = \frac{v^{3/2}}{3/2}\cdot 2 = (u^2-1)^{3/2}\cdot\frac{2}{3}\cdot 2 + C$

let $v = u^2-1$

$dv = 2u\,du$

$\frac{dv}{2} = u\,du$

$$\int \frac{1}{x\ln x}\,dx = \int \frac{1}{u}\,du = \ln(u) = \ln(\ln x) + c.$$

let $u = \ln x$
$du = \frac{1}{x}dx$

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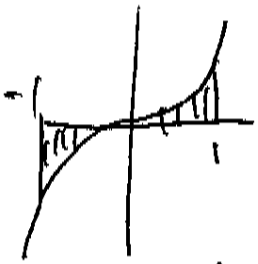
1 $\displaystyle\int_0^{10} x\,dx = \left.\frac{x^2}{2}\right|_0^{10} = \frac{100}{2} - \frac{0}{2} = 50$

2. $\displaystyle\int_0^1 x^{100}\,dx = \left.\frac{x^{101}}{101}\right|_0^1 = \frac{1^{101}}{100} - \frac{0^{101}}{100} = \frac{1}{100}.$

3. $\displaystyle\int_{-1}^{1} x^3\,dx =$  $\displaystyle = \left.\frac{x^4}{4}\right|_{-1}^{1} = \frac{1^4}{4} - \frac{(-1)^4}{4} = \underline{\underline{0}}$

same area!

[area below x-axis is negative]

4. $\displaystyle\int_0^3 e^x\,dx = \left.e^x\right|_0^3 = e^3 - e^0 = e^3 - 1$

5. $\displaystyle\int_1^e \frac{1}{x}\,dx = \left.\ln x\right|_1^e = \ln e - \ln 1 = 1 - 0 = 1.$

$$\int_2^3 x(x^2-2)^{1/2} = \int_{x=2} \frac{du}{2} \cdot u^{1/2} = \frac{1}{2} \frac{u^{3/2}}{3/2} \Big|_2^3$$

let $u = x^2-2$
$du = 2x\,dx$

$$= \frac{1}{3}\left[x^2-2\right]^{3/2} \Big|_2^3$$

$$= \frac{1}{3}\left[7^{3/2} - \cancel{2}\,2^{3/2}\right] = \frac{1}{3}\left[7^{3/2} - 2^{3/2}\right]$$

2. $$\int_1^3 \frac{x}{x^2+2}\,dx = \int_1^3 \frac{\frac{du}{2}}{u} = \frac{\ln u}{2}\Big|_1^3 = \frac{\ln(x^2+2)}{2}\Big|_1^3$$

let $u = x^2+2$
$du = 2x\,dx$

$$= \frac{\ln[9+2]}{2} - \frac{\ln[1+2]}{2}$$

$$= \frac{\ln 11}{2} - \frac{\ln 3}{2}$$

3. $$\int_1^4 (16x^2 - 16x + 15)\,dx = \frac{16x^3}{3} - \frac{16x^2}{2} + 15x \Big|_1^4$$

$$= \frac{16\cdot 4^3}{3} - \frac{16\cdot 4^2}{2} + 15\cdot 4 - \left[\frac{16\cdot 1^3}{3} - \frac{16\cdot 1^2}{2} + 15\right]$$

$$= \frac{16\cdot 64}{3} - \frac{16\cdot 256}{3} + 60 - \frac{16}{3} + 8 - 15 = \frac{?}{\text{Calculator.}}$$

1) $A = \pi r^2$ so $\frac{dA}{dt} = \pi \cdot 2r \frac{dr}{dt}$

if $\frac{dr}{dt} = 2$, $r = 1$ then $\frac{dA}{dt} = \pi \cdot 2 \cdot 1 \cdot 2 = 4\pi$.

2) if $V = \frac{4}{3}\pi r^3$ then $\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$

if $\frac{dV}{dt} = 1$, $r = 1$ then $1 = \frac{4}{3}\pi \cdot 3 \cdot 1^2 \frac{dr}{dt}$

$1 = 4\pi \frac{dr}{dt}$ so $\boxed{\frac{dr}{dt} = \frac{1}{4\pi}}$

3) $pq = 27$ $\quad \eta = \frac{-p}{q} \cdot \frac{dq}{dp} = \frac{-p}{q} \cdot \left(\frac{-q}{p}\right) = 1$.

$\frac{d}{dp}[pq] = \frac{d}{dp}[27]$

$q + p\frac{dq}{dp} = 0$

$\frac{dq}{dp} = \frac{-q}{p}$

b) Trick question: we have $pq = 27$ so R is constant!

4) $R' = 23 + 20x$

$R(x) = \int R'(x) = 23x + 20x^2 + C$

$R(0) = 0 \rightarrow C = 0$ so $\underline{R(x) = 23x + 20x^2}$

5 $MC = c' = 2 + \frac{3}{x^2} \quad x \ge 1$

$C(10) = 10$ find $C(x)$.

$$C(x) = \int c'(x)\,dx = \int \left(2 + \frac{3}{x^2}\right)dx = 2x + \frac{3 \cdot x^{-1}}{-1} + C$$

$$= 2x - \frac{3}{x} + C$$

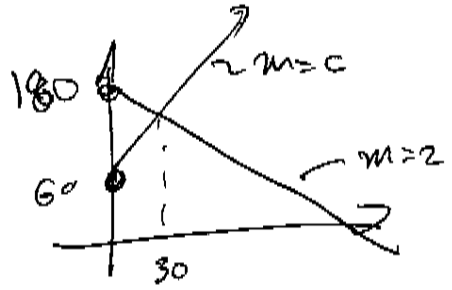
$C(10) = 20 - \frac{3}{10} + C = 10 \quad$ so $C = 10 - 20 + \frac{3}{10} = \underline{10.3}$

$$\underline{C(x) = 2x - \frac{3}{x} + 10.3}$$

6. $\overline{MR} = \overline{MC}$ when $6x + 60 = 180 - 2x$

$4x = 120$

or $\boxed{x = 30}$



$\overline{MR}-\overline{MC}$	+	0	−	
$P(x)$	↗		↘	local max

b) $R(x) = \int (180 - 2x) = 180x - \frac{2x^2}{2} + C$

$= 180x - x^2$ as $C = 0$ $(R(0) = 0)$

$$C(x) = \int (6x + 60)\,dx = \frac{6x^2}{2} + 60x = \frac{3x^2}{2} + 60x + C$$

$C(10) = 1000 = \frac{3 \cdot 100}{2} + 60 \cdot 10 + C$

$1000 = 750 + C$ so $C = 250$ and $C(x) = \frac{3x^2}{2} + 60x + 250$

$P(x) = 180 - x^2 - \left[\frac{3x^2}{2} + 60x + 250\right] = -\frac{5}{2}x^2 - 60x - 70$ & $P(x) < 0$