This test will cover the following topics:

1. The exponential and log functions – properties
2. The exponential and log functions – derivatives
3. The relation between $f(x)$ and $f'(x)$: increasing, decreasing functions and the first derivative test.
4. Critical points.
5. The relation between $f(x)$ and $f''(x)$: concavity and the second derivative test.
6. Inflection points
7. Applications of finding absolute maxima and absolute minima
8. Applications to curve sketching
9. The implicit differentiation technique.

(We will be tested on related rates on test III.)

As with our review for test I, we will work as a class on the following questions for most of the class. At the end, I will go over the answers. For your convenience, I will try to post these answers on the course website: `http://www.math.csi.cuny.edu/verzani/classes/MTH221`.

1. Find the following
$$e^{1/2}, \quad \ln(123), \quad \log_{10}(123), \quad \log_5(123)$$
2. Simplify the following using the rules of exponentials and logs. (Combine powers if possible, write logs as sums or differences.)
$$\frac{e^{3x}e^{-5x}}{e^{2x}}, \quad \frac{e^{3x}+e^{5x}}{e^{3x}}, \quad \ln(\frac{x(x-3)}{x-5}), \quad \log_5(x^2(x-3)^4)$$
3. Use the fact that logs and exponentials "undo each other" to simplify
$$\ln(e^3), \quad \ln(e^{x^2}), \quad e^{\ln 5}, \quad e^{2\ln x}$$
4. Find the first derivative of each of these functions
$$f(x)=e^{-x^2/2}, \quad f(x)=xe^{-2x}, \quad f(x)=\ln(x(x-1)(x-2)(x-3)), \quad f(x)=\ln(\sqrt{x-2})$$
5. For the function $f(x)=xe^{-x}$ do:
   (a) Find any critical points
   (b) Make a sign diagram of $f'(x)$
   (c) Classify the critical points as relative max., relative min. or neither using the first derivative test.

(d) Find the absolute maximum of this function on the interval $[0,2]$.

(e) Find the absolute maximum of this function on the interval $[2,10]$.

6. For the function $f(x) = e^{-x^2}$ do:

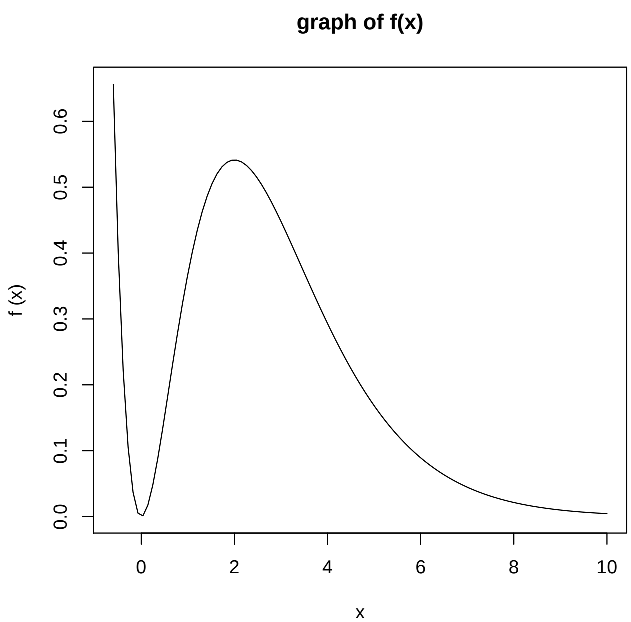
   (a) Find $f'(x)$ and $f''(x)$.

   (b) Find the critical points of $f(x)$.

   (c) Make sign diagrams of both $f'(x)$ and $f''(x)$.

   (d) Find any inflection points of $f(x)$.

7. The graph shows a function $f(x)$. Based only on this graph, produce sign diagrams of both



$f'(x)$ and $f''(x)$.

8. Suppose you know the following about $f(x)$:

   (a) It has the following limits
   $$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 1,$$

   (b) It has vertical asymptotes at 5 and 9

   (c) It has x-intercepts at 2 and 4

   (d) It has y-intercepts at 1/2

   (e) It has sign diagram for $f'(x)$:

```
f'(x)    - - 0 + * - 0 + * -  -
         1 2 3 4 5 6 7 8 9 10 11
```

   (f) It has sign diagram for $f''(x)$:

```
f''(x)   - -  0  + + * + + + * +
         1 2 2.5 3 4 5 6 7 8 9 10
```

Sketch the graph of $f(x)$.

9. A business has an advertising campaign. They measure sales as a function of t, the time since the ad campaign ran. They model the sales by

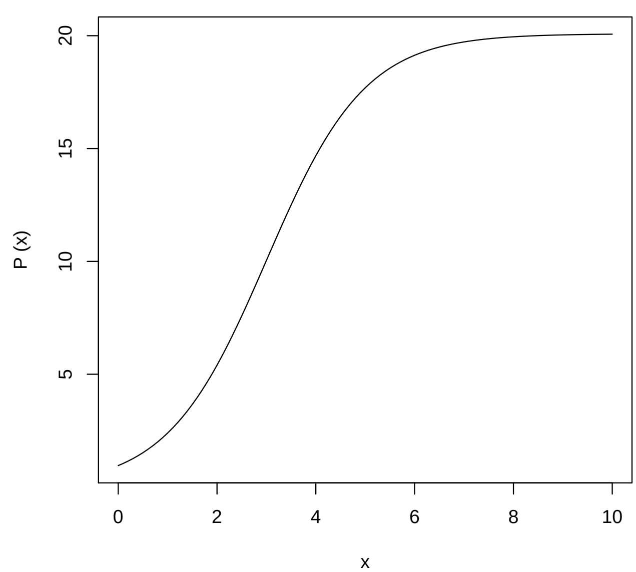
$$S(t) = (t/2)e^{-t/2}, \quad t \geq 0$$

At what time will the sales be maximal?

10. A company models profits as a function of investment, x, by the function

$$P(x) = \frac{e^x}{1+e^{x-3}}.$$

For $x \geq 0$ its graph is given here



Use the graph to answer the following:

(a) Find the sign diagram of $P''(x)$.

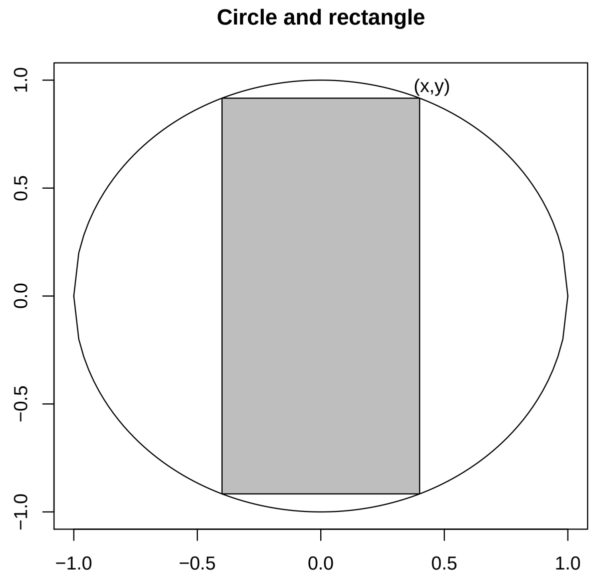
(b) What is the point of diminishing returns?

11. A company's production p is related to the number of man-hours x and the amount of investment y by

$$p^2 = 100x^{1/3}y^{2/3}$$

Suppose, p is fixed at 10 (so that $10^2 = 100x^{1/3}y^{2/3}$), find the rate of change of y with respect to x at the point $(1,1)$ using implicit differentiation.

12. A circle centered at the origin of radius 1 is to be inscribed by a rectangle. Find the rectangle



with maximum area.

Use the fact that (x,y) satisfies $x^2+y^2=1$, and $A=4xy$ to write A as a function of x alone. Then maximize this function.

13. Fill in the blank.

    (a) If $f''(x)$ changes sign at $x=c$ $(c,f(c))$ is called a(n) ________.
    (b) If $f'(x)$ changes sign at $x=c$ $(c,f(c))$ is called a(n) ________.
    (c) If $f'(c)=0$ or $f'(c)$ does not exist, then $(c,f(c))$ is called a(n) ________.

14. For each of the following sketch a curve satisfying:

    (a) $f'(x)>0$ and $f''(x)>0$.
    (b) $f'(x)>0$ and $f''(x)<0$.
    (c) $f''(x)=0$ and $f'(x)>0$.