# 221 Review answers

1. $e^{1/2}$ $\exp(1/2) = 1.64\ldots$

$\ln(123) = 4.81\ldots$

$\log_{10}(123) = 2089\ldots$

$\log_5(123) = \log_{10}123/\log_{10}5 = 2.98\ldots$

2. $\frac{e^{3x}e^{-5x}}{e^{2x}} = e^{3x-5x-2x} = e^{-4x} = \frac{1}{e^{4x}}$

$\frac{e^{3x}+e^{5x}}{e^{3x}} = \frac{e^{3x}}{e^{3x}} + \frac{e^{5x}}{e^{3x}} = 1+e^{2x}$

$\ln\frac{x(x-3)}{x-5} = \ln x + \ln(x-3) - \ln(x-5)$

$\log_5(x^2(x-3)^4) = \log_5 x^2 + \log_5(x-3)^4$

$= 2\log_5 x + 4\log_5(x-3)$

3. $\ln e^3 = 3$, $\ln e^{x^2} = x^2$, $e^{\ln 5} = 5$

$e^{2\ln x} = e^{\ln x^2} = x^2$.

$f(x) = e^{-x^2/2}$ $f' = e^{-x^2/2} \cdot (-2x/2) = \overset{-2-}{-x}e^{-x^2/2}$

$f(x) = xe^{-2x}$ $f' = 1[e^{-2x}] + x[-2e^{-2x}]$

$f(x) = \ln(x(x-1)(x-3)) = \ln x + \ln(x-1) + \ln(x-3)$

$$f' = \frac{1}{x} + \frac{1}{x-1} + \frac{1}{x-3}$$

$f(x) = \ln\sqrt{x-2} = \ln(x-2)^{1/2} = \frac{1}{2}\ln(x-2)$

$$f' = \frac{1}{2}\,\frac{1}{x-2}$$

5. $f = xe^{-x}$

$f' = 1e^{-x} + x(-e^{-x}) = e^{-x}(1-x)$

$f'$ + 0 −

1

a) c=1 is a c.p. b) c) 1 is a rel. max ↗ ↘

d) on [0,2]

$f(0) = 0$ $f(2) = \frac{2}{e^2} < f(1) = e^{-1}$

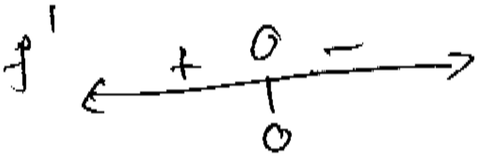
e) on [2,10] no c.p. and $f(2) > f(10)$. $\boxed{x=2}$

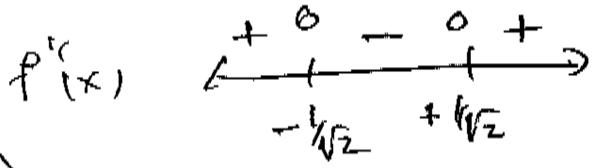
6. $f(x) = e^{-x^2}$

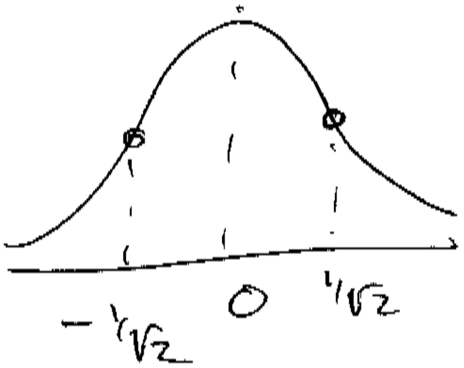
$f'(x) = e^{-x^2} \cdot (-2x)$

$f''(x) = -2(e^{-x^2}) + (-2x)\cdot(-2xe^{-x^2}) = 2e^{-x^2}[1-2x^2]$

$= -2e^{-x^2} + 4x^2e^{-x^2}$







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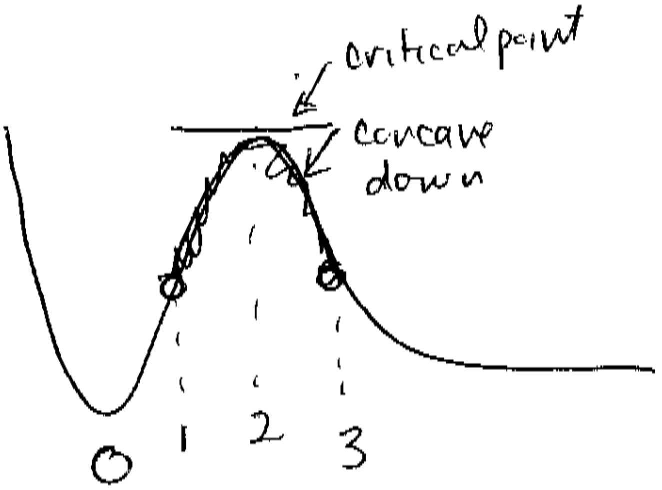
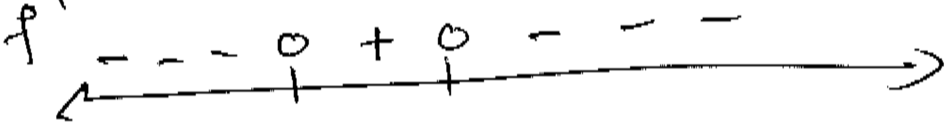
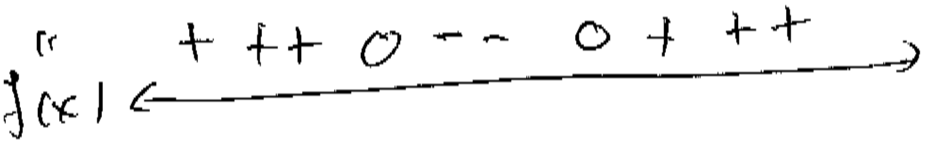
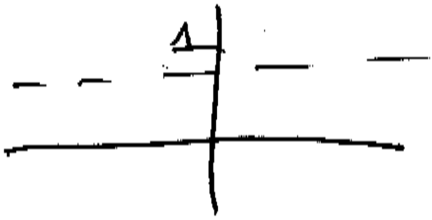




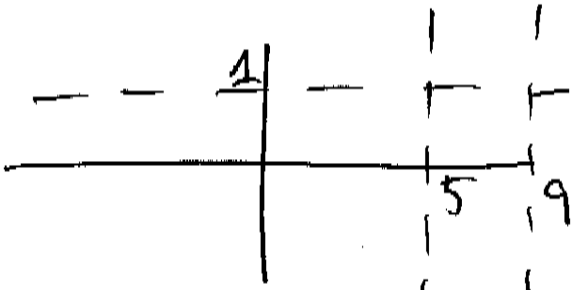


8. a)



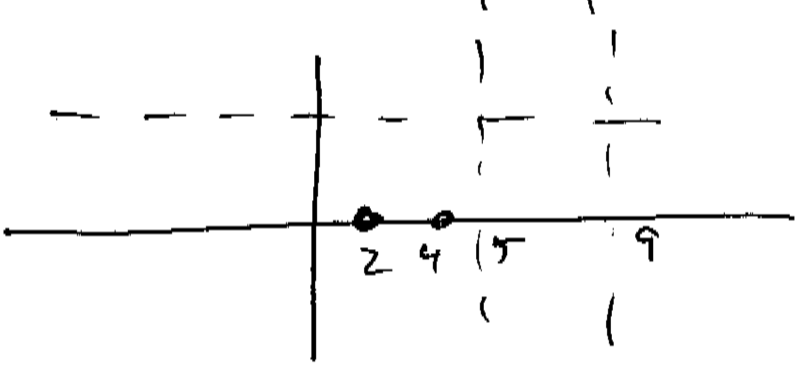
HA.

b)



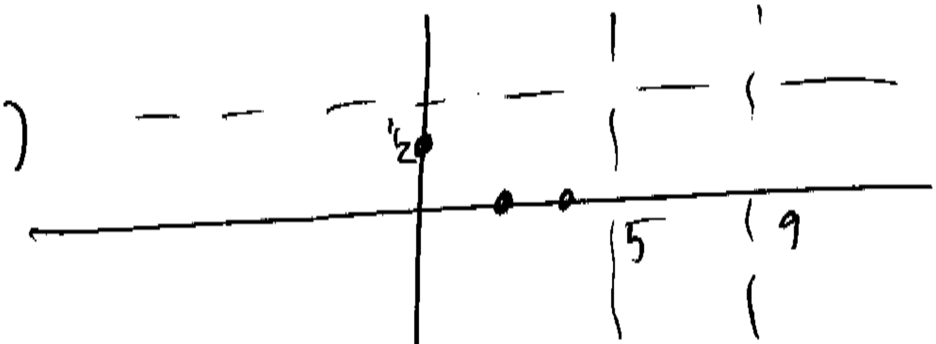
VA

c)



x-int.

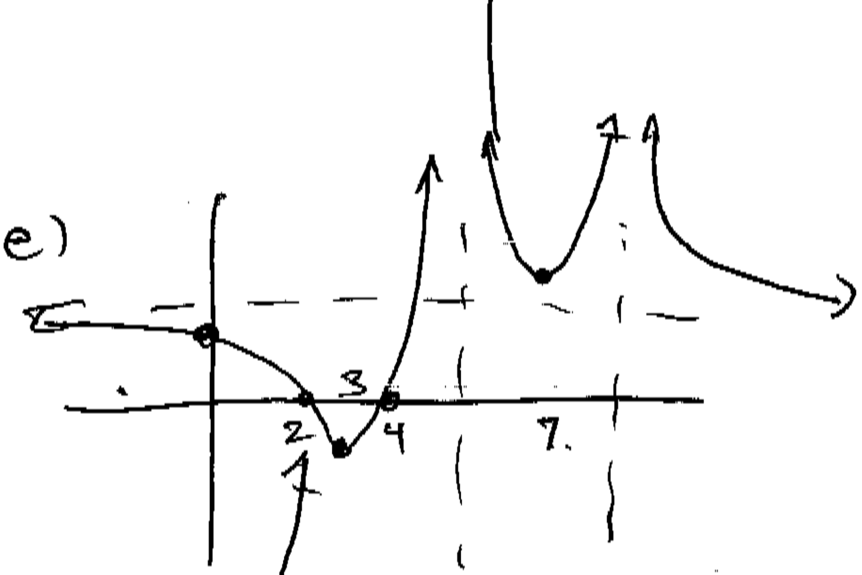
d)



y-intercept

3 is rel min
7 is rel min

e)



f) IP at 2 5.

9. Solve $S'(t) = 0$. notice $S(0) = 0$

$\lim_{t \to \infty} s(t) = 0$ so max at $s'(t) = 0$

$$s' = \frac{1}{2} \cdot e^{-t/2} + \frac{t}{2} \cdot e^{-t/2} \cdot (-\tfrac{1}{2}) = e^{-\frac{t}{2}}\left[\frac{1}{2} - \frac{t}{4}\right]$$

$s' = 0$ at $t = 2$. By first derivative test its a rel. max.

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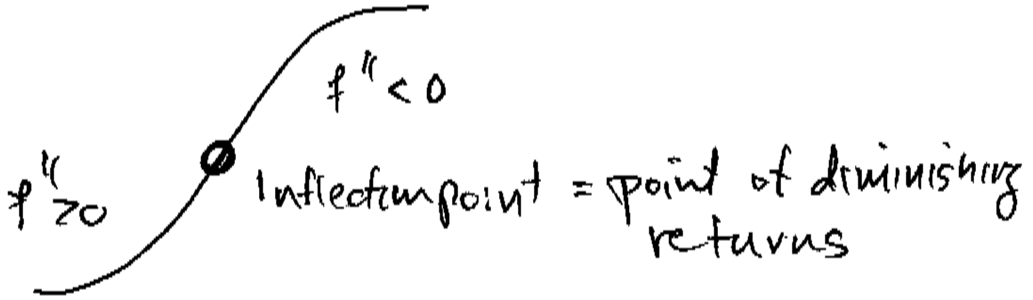


11. $100 = 100\, x^{1/3} y^{2/3}$ at $(1,1)$ $\frac{dy}{dx}$ is found by implicit diff.

$$\frac{d}{dx}[100] = \frac{d}{dx}\left[100\, x^{1/3} y^{2/3}\right]$$

$$0 = 100\left[\frac{1}{3} x^{-2/3} \cdot y^{2/3} + x^{1/3} \cdot \frac{2}{3} y^{-1/3} \cdot\right] \frac{dy}{dx}$$

$$= 100 \cdot \left[\frac{1}{3}\left[\frac{y}{x}\right]^{2/3} + \frac{2}{3}\left[\frac{x}{y}\right]^{1/3} \cdot \frac{dy}{dx}\right.$$

plug in $(1,1)$

$$0 = 100\left[\frac{1}{3} \cdot 1 + \frac{2}{3} \cdot 1 \cdot \frac{dy}{dx}\right]$$

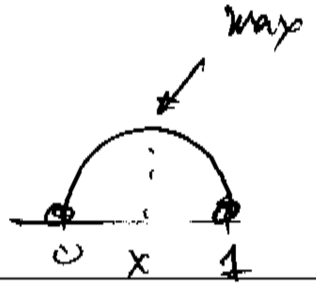
$$-\frac{1}{3} \cdot \frac{3}{2} = \frac{dy}{dx} \quad \text{or} \quad \frac{dy}{dx} = -\frac{1}{2}$$

12. fa point (x,y) we have

$$x^2+y^2=1$$

and Area = $(2x)(2y)=4xy$

Solving $A = A(x) = 4x\sqrt{1-x^2}$ w/ graph



find max by solving $A'(x)=0$.

$$A'(x) = 4\cdot\sqrt{1-x^2} + 4x\cdot\frac{1}{2}\,\frac{1}{\sqrt{1-x^2}}\cdot(-2x)$$

$$= \frac{4(1-x^2)}{\sqrt{1-x^2}} + \frac{-4x^2}{\sqrt{1-x^2}} = \frac{4(1-2x^2)}{\sqrt{1-x^2}}$$

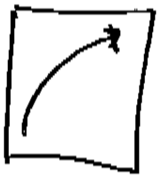
$A'(x)=0$ if $1-2x^2=0$ or $x=\pm\sqrt{\frac{1}{2}}$

hence $x=+\sqrt{\frac{1}{2}}$ so $y=+\sqrt{\frac{1}{2}}$ too and area is maximal for a square.

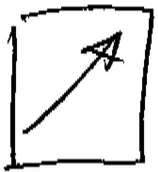
13. a) inflection point
    b) relative max (or min)
    c) critical point

14. a) ↗ ∪ ⇒

b) ↗ ∩ ⇒



c) ↗ —



(a line)