

The data file `tuna.txt` is a table of data for southern bluefin tuna found at <http://www.ccsbt.org/docs/data.html>. It can be read in as follows (cross fingers here)

```
> f = "http://www.math.csi.cuny.edu/verzani/classes/MTH804/computer/tuna.txt"
> tuna = read.table(file=url(f))
```

The variable `tuna` contains estimates for the global catch of southern bluefin tuna. We want to model the total estimate against time.

A deterministic model used by Myers and Worm (Nature 423, 280-283 (2003)) to model decline of predatory fish of species i is

$$N_i(t) = N_i(0)((1 - \delta_i)e^{-r_i t} + \delta_i).$$

For a single species the subscripts i can be dropped.

Suppose the process has observation uncertainty of the type $N_{obs,t} = N_t + W_t$ where W_t is assumed to be normal with mean 0 and unknown variance.

1. Plot the data. Do all the data points appear to be valid? If not, argue that they can be dropped.
2. Write down the negative log likelihood function for the model. Assume, you know N_0 from the graph.
3. Use the method of non-linear least squares to estimate the parameters r and d .
4. Look at the negative log likelihood as a function of σ . Can you minimize this function as a function of δ, r ? What equation do these satisfy at $\hat{\sigma}^2$?
5. Can you compare the submodel $N(0)\delta = 20000$ to the model δ is in $[0, 1]$ using the likelihood ratio test? What about the AIC criteria.