Tests of hypothesis involve a few steps:

Formulating the hypotheses We formulate hypotheses about unknown population parameters such as μ or p. We have a **null hypothesis** which is usually the hypothesis that nothing is different, and an **alternative hypothesis** which is often what we want to establish

Find a test statistic Depending on our assumptions on the data, various test statistics are appropriate. For us we have three to consider at this point.

If the data is normal with unknown mean and **known** variance then the test statistic is

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

and this has a standard normal distribution

If the data is normal with **unknown mean and variance** then the test statistic is

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

This has a t-distribution with $n-1$ degrees of freedom

If the data are simply frequencies such as Y the number of successes and N the number of trials, then let $\hat{p} = Y/N$ and then the test statistic is

$$Z = \frac{\hat{p} - p}{\sqrt{\hat{p}(1-\hat{p})/n}}$$

This has a standard normal distribution (approximately) if N is big enough

A p-value This is found by computing the probability of **observing** the test statistics value or **worse** under the H_0. The term "worse" means values which are more consistent with the alternative hypothesis.

Our goal is to find the p-value and then if desired accept or reject the null hypothesis. Remember small p-values indicate that the null hypothesis is not correct.

1 Using the computer

The computer has two helpful functions to do the above tests is you have all the data. These are `t.test()` and `prop.test()`. Two examples

A survey A survey of 400 people find 195 who support the president. Is this consistent with the notion that the president has 50% support or does it indicate his support is less?

That is, test the hypotheses

$$H_0 : p = .5, \quad H_A : p < .5$$

This is done with the function `prop.test` as follows. We input the number who agree, the total number, the null with `p=.5` and the alternative, specified as `alt="less"`.

```
> prop.test(195, 400, p = .5,  alt="less")

        1-sample proportions test with continuity correction

data:  195 out of 400, null probability 0.5
X-squared = 0.2025, df = 1, p-value = 0.3264
alternative hypothesis: true p is less than 0.5
95 percent confidence interval:
 0.0000 0.5298
sample estimates:
     p
0.4875
```

The *p*-value if there as 0.3264. This is not so small as to make the null hypotheses implausible.

Text book amounts Ten students spend the following on text books

`364 327 241 343 271 330 329 297 277 375`

Is this consistent with the fact that the average amount is 250 or indicative that it is higher?

This a test of

$$H_0 : \mu = 250, \quad H_A : \mu > 250$$

For this we use `t.test()` which needs the data which we store in the variable `x`, the null which we specify with `mu=250` and the alternative specified with `alt="greater"`.

```
> x = c(364,327,241,343,271,330,329,297,277,375)
> t.test(x,mu=250,alt="greater")

        One Sample t-test

data:  x
t = 4.826, df = 9, p-value = 0.0004696
alternative hypothesis: true mean is greater than 250
95 percent confidence interval:
 290.6   Inf
sample estimates:
mean of x
    315.4
```

Notice the *p*-value is *tiny*. We would reject the null hypothesis in this case.

Aids awareness 150 people were asked to answer True or False to the statement

> The number of people in the world who died with aids in 2003 is 3 million.

Only 55 correctly said "yes". Is this consistent with the idea that half the people are aware of the number of aids deaths, or indicative that this isn't so?

A significance test of

$$H_0 : p = .5, \quad H_A : p \neq .5$$

is asked for. This is done by specifying that the alternative is two sided as follows:

```
> prop.test(55,150,p = .5, alt="two.sided")

        1-sample proportions test with continuity correction

data:  55 out of 150, null probability 0.5
X-squared = 10.14, df = 1, p-value = 0.001451
alternative hypothesis: true p is not equal to 0.5
95 percent confidence interval:
 0.2907 0.4496
sample estimates:
     p
0.3667
```

The small *p*-value (0.001451) would lead one to reject the null hypothesis.

2 problems

For each, write down the null and alternative hypotheses, the command you used and the *p*-value found.

2.1 The heart rate of 10 smokers are reported here

```
76 79 65 85 73 91 81 77 84 72
```

Is this consistent with the mean rate of 75 or indicative that the mean rate is higher than 75?

2.2 Class grades for a small section of MTH 102 are

```
2.7 2.7 3.0 3.3 2.0 0.0 3.8 3.3 3.7 2.7 3.0
```

Are these consistent with the average grade being a 3.0?

2.3 Historically, it takes students 5 minutes to park on CSI. A survey of 10 students found that it took this long

```
6  9  8  6  9 11  3  3  7  9
```

Do a significance test to see if the amount of time it takes to park is more than 5 minutes.

2.4 Nationwide it is thought that 25 percent of college students have tatoos. A class survey of 30 found that only 4 had a tatoo. Is this consistent with the nationwide percentage or indicative that the percentage is less.

2.5 50,000 people are surveyed and 12.1 percent are found to be in poverty. Is this consistent with the assumed percentage of 11.7 percent or indicative that the rate is higher?