1. Do the following series converge? Explain your answer, and give the full details of any test to use. (For example, for the comparison test, say what series you're comparing to. For the ratio or root test, give the value of the limit you compute when applying the test.)

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{n^n}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1}\sqrt{n^2+2}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+4n}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{n-2}{2n+3}$

2. For which values of x does the following power series converge?

   $$\sum_{n=1}^{\infty} \frac{n^2}{5^n}(x-8)^n$$

3. Consider the function $f(x) = \sqrt{4+x}$. Its first four derivatives are:

   $$\begin{aligned} f'(x) &= \frac{1}{2(4+x)^{1/2}}, \\ f''(x) &= -\frac{1}{4(4+x)^{3/2}}, \\ f^{(3)}(x) &= \frac{3}{8(4+x)^{5/2}}, \\ f^{(4)}(x) &= -\frac{15}{16(4+x)^{7/2}}, \end{aligned}$$

   and the degree 3 Taylor polynomial $f(x)$ centered at 5 is

   $$T_3(x) = 3 + \frac{1}{6}(x-5) - \frac{1}{216}(x-5)^2 + \frac{1}{3888(x-5)^3}.$$

   According to the Taylor polynomial error bound, what is the largest $|f(5.9) - T_3(5.9)|$ could be?

4. Let $f(x) = 2x^{1/5}$.

   (a) Compute $f'(x)$, $f''(x)$, and $f'''(x)$.

(b) What is the 3rd degree Taylor polynomial for $f(x)$ centered at 1?

5. Find the 4th degree Taylor polynomial for $\ln(x)$ centered at 2.

6. Find the Taylor series for $f(x) = 1/x$ centered at -3.

7. How accurate is the following estimate?

$$\cos(0.3) \approx 1 - \frac{(0.3)^2}{2!} + \frac{(0.3)^4}{4!}$$

To answer the question, provide a bound on how far the right-hand side is from the left-hand side using the Taylor polynomial error bound.

8. For which values of x do the following sequences converge:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n} x^n$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\pi^n x^n}{n^\pi}$

9. Is the following statement true or false? If false, explain what is incorrect about it.

The series $\displaystyle\sum_{n=0}^{\infty} a_n$ converges if $\lim_{n\to\infty} a_n = 0$ and diverges otherwise.

10. Find the Taylor series for the following function (you can choose the center): $f(x) = xe^{-x^2}$.

11. Let $f(x) = 1/(1+x^2)$.

(a) What is the Taylor series for $f(x)$ centered at 0? What is its interval of convergence?

(b) Find $f'(x)$.

(c) Give the Taylor series for $f'(x)$ centered at 0. What is its radius of convergence?