Math 232, Exam 3 Name: _____
November 20, 2023

I pledge that I have neither given nor received unauthorized assistance during this examination.

Signature:

- **DON'T PANIC!** If you get stuck, take a deep breath and go on to the next question.
- Unless the problem says otherwise **you must show your work** sufficiently much that it's clear to me how you arrived at your answer.
- You may use a scientific calculator, but not a graphing calculator or phone.
- It is okay to leave a numerical answer like $\frac{39}{2} - (18 - e^2)$ unsimplified.
- You may bring a two-sided sheet of notes on letter-sized paper in your own handwriting.
- There are 7 problems on 7 pages.

Question	Points	Score
1	12	
2	12	
3	8	
4	8	
5	6	
6	8	
7	6	
Total:	60	

Good luck!

1. Determine for which values of x the following power series converge.

[6 points] (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(2x)^n}{n}$

[6 points] (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 x^n}{n!}$

2. State whether the following series converge or diverge. Explain your answer. If you apply a test, you must give all details of the test to get full credit. (For example, for the comparison test, say what series you're comparing to.)

[4 points] (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+1}}$

[4 points] (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n}$

[4 points] (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

3. State whether the following series converge or diverge. If they converge, **compute the sum**. If not, explain why they diverge. If you apply a test, you must give all details of the test to get full credit. (For example, for the comparison test, say what series you're comparing to.)

[4 points] (a) $\displaystyle\sum_{n=1}^{\infty} e^{-2n}$

[4 points] (b) $1 - \dfrac{1}{2} + \dfrac{1}{4} - \dfrac{1}{8} + \dfrac{1}{16} - \cdots$

[8 points] 4. Find the degree 3 Taylor polynomial $T_3(x)$ for the function $f(x) = x^{1/3}$ centered at 1.

[6 points] 5. Consider the function $f(x) = e^x$. The degree 2 Taylor polynomial for $f(x)$ centered at 0 is

$$T_2(x) = 1 + x + \frac{1}{2!}x^2.$$

According to the Taylor polynomial error bound, what is the largest $|f(-0.1) - T_2(-0.1)|$ could be?

6. Find the Taylor series centered at 0 for the following functions:

[4 points] (a) $f(x) = x^2 \sin(2x^2)$

[4 points] (b) $f(x) = \dfrac{x^3}{1 - x^2}$

[6 points] 7. Find the Taylor series of $f(x) = (\sin x)^2$ centered at 0.

Hint: $f'(x) = 2\sin(x)\cos(x) = \sin(2x)$. Or equivalently, $f(x)$ is an antiderivative of $\sin(2x)$.