1. Compute the following integrals. If they are improper integrals and don't converge, say so.

   (a) $\displaystyle\int \frac{x}{\sqrt{x^2 + 9}}\, dx$

   (b) $\displaystyle\int \frac{x^2 + 5}{(x + 1)^2(x - 2)}\, dx$

   (c) $\displaystyle\int_0^1 x^2 \ln(x^3)\, dx$

   (d) $\displaystyle\int_0^\infty \frac{1}{16x^2 + 1}\, dx$

   (e) $\displaystyle\int \frac{1}{(11 - x^2)^{1/2}}\, dx$

   (f) $\displaystyle\int_{-1/2}^{1/2} \frac{1}{(2x + 1)^{1/3}}\, dx$

2. Find the volume of the sphere of radius R by rotating the semicircle bounded by $x^2 + y^2 = R^2$ about the y-axis, using cylindrical shells.

3. Consider the region in between the y-axis, the line $y = 4$, and the curve $y = x^2/4$:
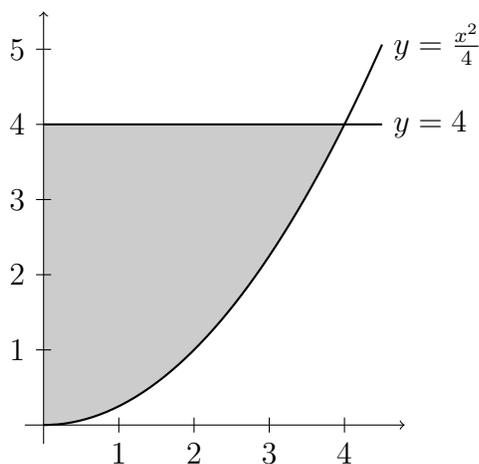
   Now revolve this region around the x-axis to form a solid.

   Set up *but do not compute* integrals to find the volume of this solid using both the disc/washer method and the cylindrical shells method.

4. Here are two questions related to trigonometric substitution:

   (a) Suppose that $\sin\theta = x$. What is $\cot\theta$? (Your answer should be in terms of x but should not involve any inverse trigonometric functions. You can assume that $0 < \theta < \pi/2$.)

So the other side of the triangle is $\sqrt{1 - x^2}$, and

$$\cot\theta = 1/\tan\theta = \frac{\sqrt{1 - x^2}}{x}.$$

(b) Suppose that $\tan\theta = x$. What is $\csc\theta$? (Your answer should be in terms of x but should not involve any inverse trigonometric functions. You can assume that $0 < \theta < \pi/2$.)

5. Compute the following sum, or say that it does not converge.

$$\sum_{n=2}^{\infty} \frac{3^{n+1}}{5^n}.$$

6. Does the series

$$\sum_{n=1}^{\infty} \frac{n^3}{n^4 + 1}$$

converge or diverge? Explain your answer. If you apply a test, you must give all details of the test to get full credit. (For example, for the comparison test, say what series you're comparing to.)

7. Does the series

$$\sum_{n=2}^{\infty} \frac{n^2 - 1}{n^2}$$

converge or diverge? Explain your answer. If you apply a test, you must give all details of the test to get full credit. (For example, for the comparison test, say what series you're comparing to.)

8. Does the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 4n}$$

converge or diverge? Explain your answer. If you apply a test, you must give all details of the test to get full credit. (For example, for the comparison test, say what series you're comparing to.)