

*I pledge that I have neither given nor received
unauthorized assistance during this examination.*

Signature:

- **DON'T PANIC!** If you get stuck, take a deep breath and go on to the next question.
- Unless the problem says otherwise **you must show your work** sufficiently much that it's clear to me how you arrived at your answer.
- No calculators or electronic devices are allowed.
- Unless the problem specifically says, you **do not** need to calculate the final answer. An answer that looks something like

$$\frac{\binom{6}{2}\binom{8}{5}}{\binom{23}{4}} \quad \text{or} \quad \frac{(12)(11)(10) + (11)(10)(9)}{14!}$$

is entirely acceptable.

- You may bring a two-sided sheet of notes on letter-sized paper in your own handwriting.
- There are 8 problems on 10 pages.

Question	Points	Score
1	18	
2	12	
3	9	
4	6	
5	11	
6	6	
7	10	
8	12	
Total:	84	

Good luck!

1. A company manufactures screws. Each screw they make is defective with probability $1/50$.

[2 points]

- (a) Let X be the number of the first defective screw. (For example, if the first three screws are good and the fourth is defective, then $X = 4$.) What is $E(X)$?

[3 points]

- (b) With X as in the last problem, what is $P(X > 25)$?

[3 points]

- (c) With X as in the last problem, what is $P(X = 10)$?

[2 points] (d) What is the distribution of X ? (Your answer should give the name and parameter of the distribution.)

[2 points] (e) Let Y be the number of defective screws in the first 200. What is $E(Y)$?

[3 points] (f) With Y as in the last problem, what is $P(Y \leq 2)$?

[3 points] (g) With Y as in the last problem, what is $P(Y = 1 \mid Y \geq 1)$?

[12 points] 2. The joint probability density function of X and Y is given by

$$f(x, y) = \begin{cases} \frac{1}{24}(2x + 3y^2) & \text{if } 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

(a) What is the marginal probability density function of X ? **Please make it clear if the density you find is only for an interval.**

(b) What is $P(X < 1 \text{ and } Y > 1)$? **Give a final, numerical answer.**

- (c) Set up an integral to find $P(Y > X^2)$. **Your answer should be a parametrized, double integral, but don't compute it.**

- (d) Set up an integral to compute $E(e^{XY})$. **Your answer should be a parametrized, double integral, but don't compute it.**

[9 points] 3. Let X have probability density function $f(x) = \frac{3}{2}x^2$ on the interval $[-1, 1]$. **In this question, compute a final numerical answer for all questions.**

(a) Find $P(X > \frac{1}{2})$.

(b) Find $E(X)$.

(c) Find $\text{Var}(X)$.

- [6 points] 4. In about 10% of all years, there is a significant earthquake in Turkey. Assuming that each year is independent, use the normal approximation to estimate the probability that over the last 10000 years, there were more than 1060 containing significant earthquakes. **Give your answer in terms of $\Phi(x)$, the cumulative distribution function of the standard normal distribution.**

5. The following table gives the joint mass function of X and Y :

		Y		
		1	2	3
X	1	.3	.2	0
	2	.2	.1	0
	3	0	.1	.1

[2 points] (a) Give the marginal probability mass functions of X and Y .

[3 points] (b) What is $P(X = 3 \mid Y = 2)$?

[3 points] (c) What is $P(X = Y + 1)$?

[3 points] (d) What is $E(XY)$?

- [6 points] 6. Let $X \sim \text{Unif}[-1, 1]$ and let $Y = X^3$. Find the probability density function of Y . **Please make it clear if the density you find is only for an interval.**

7. Let X be the number of successes in 10000 independent trials with success probability $1/20000$.

[2 points]

- (a) What is the **exact** distribution of X ? (Your answer should be a named distribution and you should give its parameter(s).)

[3 points]

- (b) What is the **exact** value of $P(X \leq 1)$? (You just need to write down an expression, not compute the numerical value.)

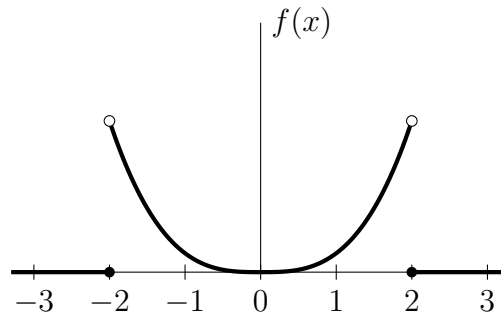
[2 points]

- (c) What distribution is appropriate to **approximate** the distribution of X ? (Your answer should be a named distribution and you should give its parameter(s).)

[3 points]

- (d) What is the **approximate** value of $P(X \leq 1)$ according to this distribution?

[12 points] 8. Random variable X has probability density function $f(x)$ depicted below:



Let $Y = X^2$. Say whether the following statements about X and Y are true or false. You don't need to explain your answers.

(a) $P(X = 1.5) = 0$ ☐ True ☐ False

(b) $P(X > 0) = 1$ ☐ True ☐ False

(c) $P(Y > 0) = 1$ ☐ True ☐ False

(d) $P(-2 < X < 2) = 1$ ☐ True ☐ False

(e) $P(-2 < Y < 2) = 1$ ☐ True ☐ False

(f) $P(0 \leq X \leq 1) < P(1 \leq X \leq 2)$ ☐ True ☐ False