

*I pledge that I have neither given nor received
unauthorized assistance during this examination.*

Signature:

- **DON'T PANIC!** If you get stuck, take a deep breath and go on to the next question.
- Unless the problem says otherwise **you must show your work** sufficiently much that it's clear to me how you arrived at your answer.
- You may use any sort of calculator, but not a phone or a computer.
- It is okay to leave a numerical answer like $\frac{39}{2} - (18 - e^2)$ unsimplified.
- You may bring a two-sided sheet of notes on letter-sized paper in your own handwriting.
- There are 7 problems on 10 pages.

Question	Points	Score
1	16	
2	12	
3	10	
4	10	
5	12	
6	8	
7	10	
Total:	78	

Good luck!

[16 points] 1. Find the derivatives of the following functions. Do **not** simplify your solutions.

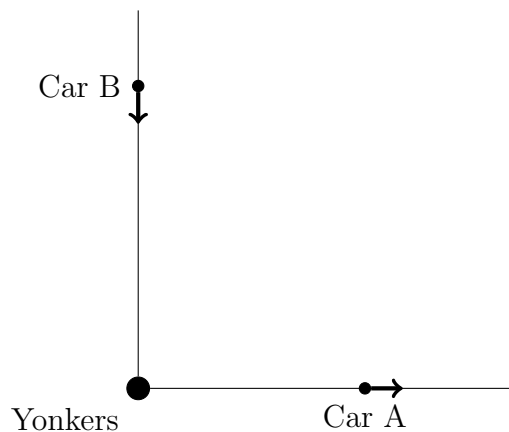
(a) $f(t) = \frac{1 + e^t}{1 - e^t}$

(b) $f(x) = \ln(4x^2 + 1)$

(c) $f(\theta) = \cos(\cos(\cos \theta))$

(d) $f(x) = \sin(2x) \cos x$

2. Car A drives east away from Yonkers at 30 mph. Car B drives south towards Yonkers at 10 mph. (See the diagram below.)



[2 points]

- (a) Let u be the distance between Car A and Car B. What is u at the moment when Car A is 3 miles east of Yonkers and Car B is 4 miles north of Yonkers?

[10 points]

- (b) What is the rate of change of u at the moment when Car A is 3 miles east of Yonkers and Car B is 4 miles north of Yonkers?

3. Suppose that the signs of $f'(x)$ and $f''(x)$ are given below:



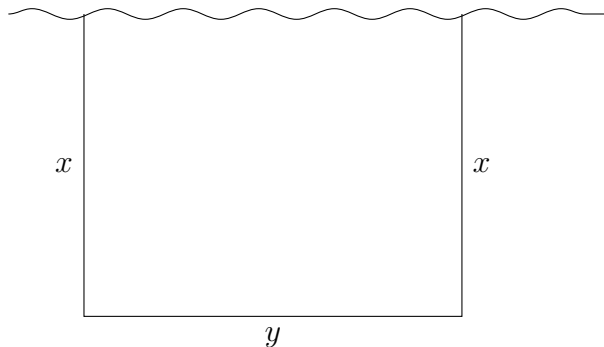
- [2 points] (a) Give all x -coordinates where $f(x)$ has a local minimum, or state that there are none.

- [2 points] (b) Give all x -coordinates where $f(x)$ has a local maximum, or state that there are none.

- [2 points] (c) Give all x -coordinates where $f(x)$ has an inflection point, or state that there are none.

- [4 points] (d) Sketch the graph in the space above the sign diagrams.

- [10 points] 4. A rectangular area along the ocean is to be fenced off for a private event. The fence only needs to cover three sides of the area; the ocean is the fourth side. Suppose the fence parallel to the water has length y and the two sides of fence perpendicular to the water have length x as in the diagram.



Suppose that you have 80 meters of fencing in total that you will use to block off the area. What are the dimensions x and y of the rectangle with the largest area satisfying this constraint?

5. Let $f(x) = \frac{1}{4 - x^2}$.

[3 points] (a) Does this function have horizontal asymptotes at $\pm\infty$? If so, say what they are.

[3 points] (b) Does this function have any vertical asymptotes? If so, say what they are.

[3 points] (c) On what intervals is this function increasing?

[3 points] (d) On what intervals is this function decreasing?

- [8 points] 6. Consider the curve formed by the solutions to $x^3 + y^3 = 3xy$. What is the slope of its tangent line at the point $(\frac{2}{3}, \frac{4}{3})$?

7. Let $f(x) = 1 + (\sin x)(\cos x)$.

[2 points]

(a) Find $f'(x)$.

[8 points]

(b) Estimate the value of $f(0.01)$ using the linear approximation of $f(x)$ based at $x = 0$.