Name: _____

> *I pledge that I have neither given nor received*
> *unauthorized assistance during this examination.*
>
>
> **Signature:**

- **DON'T PANIC!** If you get stuck, take a deep breath and go on to the next question.

- Unless the problem says otherwise **you must show your work** sufficiently much that it's clear to me how you arrived at your answer.

- You may use a scientific calculator, but not a graphing calculator or phone.

- It is okay to leave a numerical answer like $\frac{39}{2} - (18 - e^2)$ unsimplified.

- You may bring a two-sided sheet of notes on letter-sized paper in your own handwriting.

- There are 9 problems on 10 pages.

Question	Points	Score
1	8	
2	4	
3	4	
4	6	
5	6	
6	8	
7	8	
8	8	
9	6	
Total:	58	

Good luck!

1. Consider the region underneath the curve $y = 1 - x^2$ from $x = 0$ to $x = 1$, shown here:
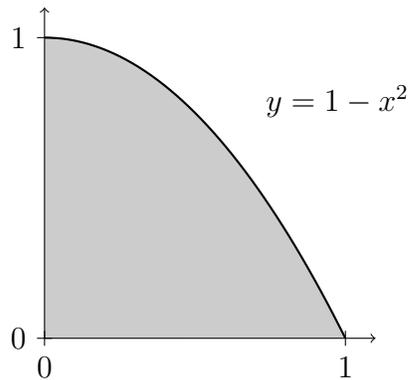
Suppose we rotate this region around the y-axis to form a solid.

[4 points]   (a) Set up **but do not compute** an integral that computes the volume of this region using the disc/washer method.

[4 points]   (b) Set up **but do not compute** an integral that computes the volume of this region using the cylindrical shell disc/washer method.

[4 points] 2. In each example, determine whether the improper integral $\int_0^\infty f(x)\,dx$ is convergent based on the sketch.

Assume that the sketch accurately represents the function, i.e., the function continues to follow whatever trend is shown on the sketch. Please note that the functions may have asymptotes both as $x \to \infty$ and as $x \to 0$. Answer **yes** if the integral is definitely convergent based on the sketch, **no** if it's definitely not, and **maybe** if the sketch doesn't give enough information to say for sure.

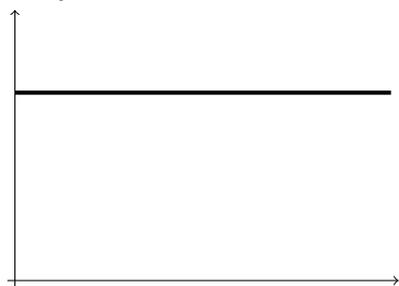(a) Is $\int_0^\infty f(x)\,dx$ convergent for the following function $f(x)$?
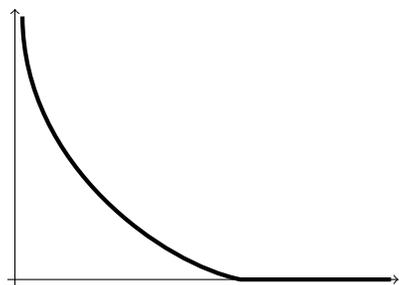
    ◯ Yes   ◯ No   ◯ Maybe

(b) Is $\int_0^\infty f(x)\,dx$ convergent for the following function $f(x)$?
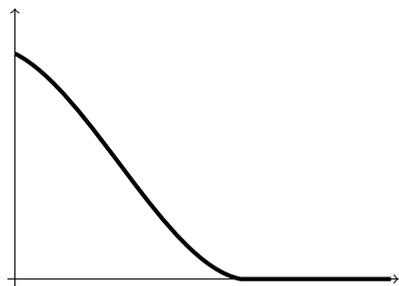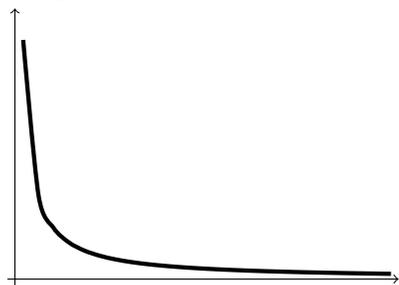
    ◯ Yes   ◯ No   ◯ Maybe

(c) Is $\int_0^\infty f(x)\,dx$ convergent for the following function $f(x)$?

    ◯ Yes   ◯ No   ◯ Maybe

(d) Is $\int_0^\infty f(x)\,dx$ convergent for the following function $f(x)$?

    ◯ Yes   ◯ No   ◯ Maybe

[4 points] 3. For the following series, compute the sum or state that it does not converge. Explain your answer. If you apply a test, you must give all details of the test to get full credit. (For example, for the comparison test, say what series you're comparing to.)

$$\sum_{n=1}^{\infty} e^{\frac{1}{n^2}}$$

[6 points] 4. Compute $\displaystyle\int_0^{\infty} e^{-3x}\, dx$ or demonstrate that the integral does not converge.

5. Consider the series $\sum\limits_{n=1}^{\infty} \dfrac{3(2^n)}{10^{n-1}}$.

[2 points]   (a) List the first three terms of this series.

[4 points]   (b) Compute the sum or state that it does not converge. Explain your answer. If you apply a test, you must give all details of the test to get full credit. (For example, for the comparison test, say what series you're comparing to.)

[8 points]   6. Find $\displaystyle\int \frac{x^2}{\sqrt{4-x^2}}\,dx$.

7. State whether the following series converge or diverge. Explain your answer. If you apply a test, you must give all details of the test to get full credit. (For example, for the comparison test, say what series you're comparing to.)

[4 points]   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{n^2 - 3}{n^4 + 3}$

[4 points]   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n - 1}{n}$

[8 points] 8. Compute $\displaystyle\int \frac{2x - 5}{(x - 2)^2(x - 3)}\, dx.$

9. Find the following antiderivatives:

[3 points]   (a) $\displaystyle\int \cos(x)\sqrt{\sin(x)}\,dx$

[3 points]   (b) $\displaystyle\int \ln(x)\,dx$

TABLE OF TRIGONOMETRIC INTEGRALS

$$\int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + C = \frac{x}{2} - \frac{1}{2} \sin x \cos x + C \qquad \boxed{3}$$

$$\int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + C = \frac{x}{2} + \frac{1}{2} \sin x \cos x + C \qquad \boxed{4}$$

$$\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx \qquad \boxed{5}$$

$$\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx \qquad \boxed{6}$$

$$\int \tan x \, dx = \ln |\sec x| + C = -\ln |\cos x| + C \qquad \boxed{7}$$

$$\int \tan^m x \, dx = \frac{\tan^{m-1} x}{m-1} - \int \tan^{m-2} x \, dx \qquad \boxed{8}$$

$$\int \cot x \, dx = -\ln |\csc x| + C = \ln |\sin x| + C \qquad \boxed{9}$$

$$\int \cot^m x \, dx = -\frac{\cot^{m-1} x}{m-1} - \int \cot^{m-2} x \, dx \qquad \boxed{10}$$

$$\int \sec x \, dx = \ln \left| \sec x + \tan x \right| + C \qquad \boxed{11}$$

$$\int \sec^m x \, dx = \frac{\tan x \sec^{m-2} x}{m-1} + \frac{m-2}{m-1} \int \sec^{m-2} x \, dx \qquad \boxed{12}$$

$$\int \csc x \, dx = \ln \left| \csc x - \cot x \right| + C \qquad \boxed{13}$$

$$\int \csc^m x \, dx = -\frac{\cot x \csc^{m-2} x}{m-1} + \frac{m-2}{m-1} \int \csc^{m-2} x \, dx \qquad \boxed{14}$$

$$\int \sin mx \sin nx \, dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + C \quad (m \neq \pm n) \qquad \boxed{15}$$

$$\int \sin mx \cos nx \, dx = -\frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)} + C \quad (m \neq \pm n) \qquad \boxed{16}$$

$$\int \cos mx \cos nx \, dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + C \quad (m \neq \pm n) \qquad \boxed{17}$$