Math 311, Midterm 1 Name: _____
October 8, 2025

I pledge that I have neither given nor received unauthorized assistance during this examination.

Signature:

- **DON'T PANIC!** If you get stuck, take a deep breath and go on to the next question.
- Unless the problem says otherwise **you must show your work** sufficiently much that it's clear to me how you arrived at your answer.
- No calculators or electronic devices are allowed.
- Unless the problem specifically says, you **do not** need to calculate the final answer. An answer that looks something like

$$\frac{\binom{6}{2}\binom{8}{5}}{\binom{23}{4}} \quad \text{or} \quad \frac{(12)(11)(10)+(11)(10)(9)}{14!}$$

  is entirely acceptable.
- You may bring a two-sided sheet of notes on letter-sized paper in your own handwriting.
- There are 7 problems on 7 pages.

Question	Points	Score
1	9	
2	6	
3	4	
4	8	
5	6	
6	8	
7	6	
Total:	47	

Good luck!

[9 points] 1. An urn contains 5 white balls, 6 black balls, and 9 red balls. You reach into the urn, grab three balls, and pull them all out.

(a) What is the probability that all three balls are white?

(b) What is the probability that exactly two of three balls are white?

(c) Conditional on at least two of the balls being white, what is the probability that exactly two are white?

[6 points] 2. Coin #1 and coin #2 are fair, but coin #3 lands on tails with probability 3/4.

You choose one of three coins uniformly at random, and you flip it and get tails. What is the probability that you chose coin #3?

[4 points] 3. For each collection of events, say whether you would expect them to be **disjoint**, **independent**, or **neither**.

(a) Three coins are flipped.

$A = \{\text{coins \#1 and \#3 are heads}\}$
$B = \{\text{coins \#1, \#2, and \#3 are tails}\}$

◯ disjoint ◯ independent ◯ neither

(b) Three coins are flipped.

$A = \{\text{coins \#1 and \#3 are heads}\}$
$B = \{\text{coins \#1, \#2, and \#3 are heads}\}$

◯ disjoint ◯ independent ◯ neither

(c) Two strangers walk into a room.

$A = \{\text{person \#1 was born on a Monday}\}$
$B = \{\text{person \#2 was born on a Tuesday}\}$

Events A and B are:
◯ disjoint ◯ independent ◯ neither

(d) One person walks into a room.

$A = \{\text{person was born on a Monday}\}$
$B = \{\text{person was born on a Tuesday}\}$

Events A and B are:
◯ disjoint ◯ independent ◯ neither

4. An urn contains balls numbered $1, \ldots, 20$. One ball is chosen, then a second, then a third, and then a fourth. Balls are not reinserted into the urn, so each ball can only be selected once.

[2 points] (a) How large is the natural sample space for this random experiment?

[3 points] (b) What is the probability that exactly two of the four chosen balls have an even number on them?

[3 points] (c) What is the probability that the first ball is even, the second is odd, the third is even, and the fourth is odd?

[6 points] 5. You roll a die five times. If you get exactly three appearances of some number and two of another, call the rolls a "full house". (For example, it's a full house if you roll 1, 3, 3, 1, 1.)

What is the probability of rolling a full house?

6. A random student is picked from a school. There is a 50% chance that the student is wearing a watch, a 40% chance the student is wearing a bracelet, and a 30% chance the student is wearing neither a watch nor a bracelet.

[3 points] (a) What is the probability the student is wearing a watch or a bracelet (or both)?

[3 points] (b) What is the probability the student is wearing a watch and a bracelet?

[2 points] (c) Are the events of wearing a watch and wearing a bracelet independent? Why or why not?

[6 points] 7. There are two kinds of employees at Lab X: technicians and scientists. The lab can only operate if it has at least two technicians and one scientist.

One a certain day, two technicians and two scientists are scheduled to work. Technicians #1 and #2 show up with probability p_1 and p_2. Scientists #1 and #2 show up with probability q_1 and q_2. The events of each employee showing up are independent. Find the probability that the lab is able to operate. (Your answer will be in terms of p_1, p_2, q_1, and q_2.)