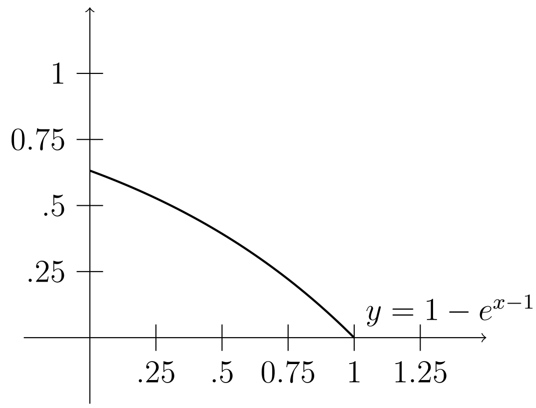
1. Here is a graph of the curve $y = 1 - e^{x-1}$ from $x = 0$ to $x = 1$:



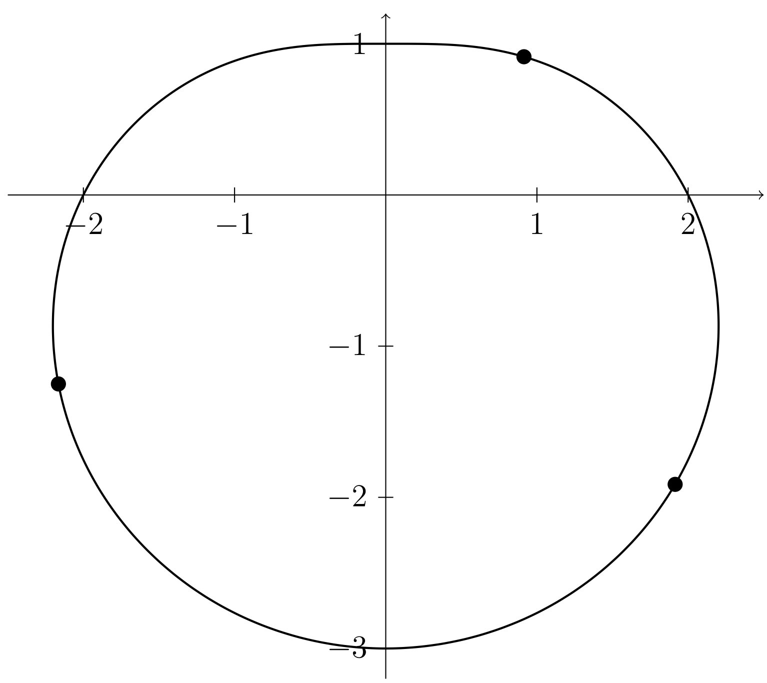
   (a) Find the area of the region under the curve.

   Now, take this region and rotate it around the y-axis to sweep out a solid of revolution.

   (b) Find the volume of this solid using the disc/washer method.

   (c) Find the volume of this solid using the cylindrical shells method.

2. Here is a graph of the polar equation $r = 2 - \sin\theta$. The points on the graph are plotted at locations $\theta = \pi/4$, $\theta = 7\pi/6$, and $\theta = 7\pi/4$.



   (a) What are the polar coordinates of the three points?

   (b) What are the rectangular coordinates of the three points? (Give exact answers, not decimal approximations; you'll need to know the special values of $\sin\theta$ and $\cos\theta$ that can be computed exactly as square roots.)

(c) Find the area enclosed by the curve.

3. Do the following series converge? Explain your answer. If you apply a test, you must give all details of the test to get full credit. (For example, for the comparison test, say what series you're comparing to. For the ratio or root test, give the value of the limit you compute when applying the test.)

   (a) $\sum_{n=1}^{\infty} \frac{n^3}{4^n}$

   (b) $\sum_{n=1}^{\infty} \frac{1+2^n+3^n}{5^n}$

   (c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{2^n+1}}$

   (d) $\sum_{n=1}^{\infty} \frac{2n+3}{3n-5}$

4. Consider the curves $y = x$ and $y = xe^{5x}$. Find the area between the two curves from $x = 0$ to $x = 2$.

5. A particle moves on a plane and is at location $\big(x(t), y(t)\big)$ where

$$x(t) = -t,$$
$$y(t) = t^2 - 5t + 2$$

   (a) Is the particle ever at position $(-1, -2)$? If so, at what time?

   (b) What is the particle's speed at time $t = -3$?

   (c) Set up but do not compute an integral that determines how far the particle moves between times 0 and 1. (If you want a challenge, also compute the integral.)

6. Say if each statement is true or false:

   (a) If $\lim_{x\to\infty} f(x) = 0$, then $\int_1^{\infty} f(x)\,dx$ converges.

   (b) If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

   (c) The technique of partial fractions can be used to find $\int \frac{1}{(x-2)(x-3)}\,dx$.

   (d) The interval $[2, 4]$ might be the interval of convergence for a power series

$$\sum_{n=0}^{\infty} a_n(x-3)^n.$$

(e) The interval $[2, 5)$ might be the interval of convergence for a power series

$$\sum_{n=0}^{\infty} a_n(x-3)^n.$$

(f) The interval $(-\infty, \infty)$ might be the interval of convergence for a power series

$$\sum_{n=0}^{\infty} a_n(x-3)^n.$$

(g) The interval $(1, 5]$ might be the interval of convergence for a power series

$$\sum_{n=0}^{\infty} a_n(x-3)^n.$$

(h) The technique of trigonometric substitution can be used to find $\int e^{-x^2}\, dx$.

7. Set up **but do not solve** integrals to find the volume of the solid obtained by revolving the region bounded between the curves $y = x^2 - 1$ and the line $y = -x + 5$ about the following lines:
   (a) $y = 10$ (use disc/washer method)
   (b) $y = -3$ (use shell method)

8. Compute the following integrals:
   (a) $\displaystyle\int_1^{e^2} \frac{4 + \ln x}{x}\, dx$
   (b) $\displaystyle\int \frac{e^t}{e^{2t} + 3e^t + 2}\, dt$
   (c) $\displaystyle\int_1^{\infty} \frac{1}{(2x+1)^3}\, dx$
   (d) $\displaystyle\int \frac{9t+1}{3t+4}\, dt$