Math 232, Final Name: ______________________________

December 18, 2023

I pledge that I have neither given nor received unauthorized assistance during this examination.

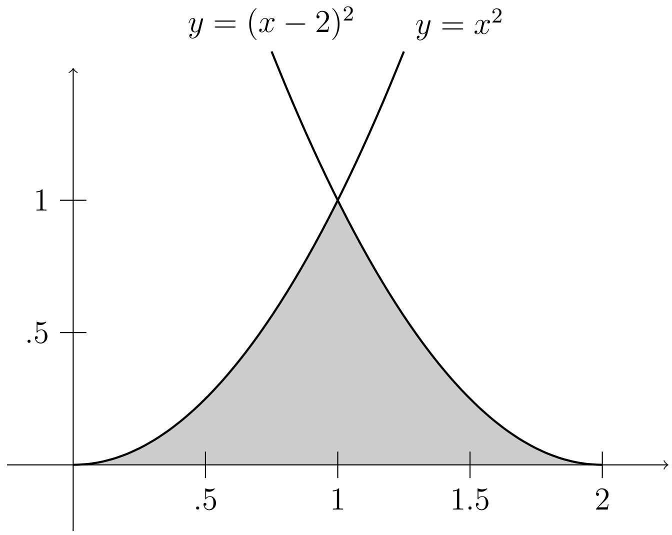
Signature:

- **DON'T PANIC!** If you get stuck, take a deep breath and go on to the next question.
- Unless the problem says otherwise **you must show your work** sufficiently much that it's clear to me how you arrived at your answer.
- You may use a scientific calculator, but not a graphing calculator or phone.
- It is okay to leave a numerical answer like $\frac{39}{2} - (18 - e^2)$ unsimplified.
- You may bring a two-sided sheet of notes on letter-sized paper in your own handwriting.
- There are 9 problems on 12 pages.

Question	Points	Score
1	10	
2	8	
3	8	
4	8	
5	8	
6	11	
7	12	
8	6	
9	6	
Total:	77	

Good luck!

[10 points] 1. A region is bounded by the x-axis and the curves $y = x^2$ and $y = (x-2)^2$:



The region is rotated around the y-axis to form a solid. (Please read carefully: **y-axis**.)

(a) Set up **but do not compute** an expression to find the volume of the solid using the disc/washer method.

(b) Set up **but do not compute** an expression to find the volume of the solid using the cylindrical shells method.

[8 points] 2. State whether the following series converge or diverge. If the series converges, **compute its sum**. If not, explain why it diverges. If you apply a test, you must give all details of the test to get full credit. (For example, for the comparison test, say what series you're comparing to.)

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{\sqrt{2^n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{2n+1}$

[8 points] 3. State whether the following series converge or diverge. Explain your answer. If you apply a test, you must give all details of the test to get full credit. (For example, for the comparison test, say what series you're comparing to.)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{5^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

[8 points] 4. Compute $\int \frac{1}{(4x^2+1)^{3/2}}\,dx$.

[8 points] 5. Find the Taylor series of the following functions centered at 0. You do not need to determine the interval of convergence.

(a) $f(x) = xe^{-x^2}$

(b) $f(x) = \dfrac{x}{1-2x}$

6. A particle at time t is at position $\big(x(t),\, y(t)\big)$ where

$$x(t) = \sin(2t)\cos(t),$$
$$y(t) = \sin(2t)\sin(t).$$

[3 points] (a) At time 0, the particle is at position $(0, 0)$. When is the next time that it returns to $(0, 0)$?

[4 points] (b) The particle sketches out a curve. What is the slope of the tangent line to this curve at the particle's location at time $\pi/4$?

[4 points] (c) Set up **but do not compute** an integral to find the total distance traveled by the particle from time 0 to time $\pi/2$.

[12 points] 7. Compute the following integrals. If they do not converge, say so and explain why.

(a) $\displaystyle\int_2^\infty \frac{1}{x^2}\,dx$

(b) $\displaystyle\int \frac{x+1}{(x-1)^2}\,dx$

(c) $\displaystyle\int x\ln x\,dx$

[6 points] 8. (a) Convert the point $(x, y) = (-2\sqrt{3}, 2)$ to polar coordinates (r, θ).

(b) Convert the point $(r, \theta) = (12, -\pi/4)$ to rectangular coordinates (x, y).

[6 points] 9. For which values of x does the following power series converge?

$$\sum_{n=0}^{\infty} \frac{nx^n}{2^n}$$

TABLE OF TRIGONOMETRIC INTEGRALS

$$\int \sin^2 x\,dx = \frac{x}{2} - \frac{\sin 2x}{4} + C = \frac{x}{2} - \frac{1}{2}\sin x \cos x + C \qquad \boxed{3}$$

$$\int \cos^2 x\,dx = \frac{x}{2} + \frac{\sin 2x}{4} + C = \frac{x}{2} + \frac{1}{2}\sin x \cos x + C \qquad \boxed{4}$$

$$\int \sin^n x\,dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx \qquad \boxed{5}$$

$$\int \cos^n x\,dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx \qquad \boxed{6}$$

$$\int \tan x\,dx = \ln|\sec x| + C = -\ln|\cos x| + C \qquad \boxed{7}$$

$$\int \tan^m x\,dx = \frac{\tan^{m-1} x}{m-1} - \int \tan^{m-2} x\,dx \qquad \boxed{8}$$

$$\int \cot x\,dx = -\ln|\csc x| + C = \ln|\sin x| + C \qquad \boxed{9}$$

$$\int \cot^m x\,dx = -\frac{\cot^{m-1} x}{m-1} - \int \cot^{m-2} x\,dx \qquad \boxed{10}$$

$$\int \sec x\,dx = \ln\left|\sec x + \tan x\right| + C \qquad \boxed{11}$$

$$\int \sec^m x\,dx = \frac{\tan x \sec^{m-2} x}{m-1} + \frac{m-2}{m-1}\int \sec^{m-2} x\,dx \qquad \boxed{12}$$

$$\int \csc x\,dx = \ln\left|\csc x - \cot x\right| + C \qquad \boxed{13}$$

$$\int \csc^m x\,dx = -\frac{\cot x \csc^{m-2} x}{m-1} + \frac{m-2}{m-1}\int \csc^{m-2} x\,dx \qquad \boxed{14}$$

$$\int \sin mx \sin nx\,dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + C \quad (m \neq \pm n) \qquad \boxed{15}$$

$$\int \sin mx \cos nx\,dx = -\frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)} + C \quad (m \neq \pm n) \qquad \boxed{16}$$

$$\int \cos mx \cos nx\,dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + C \quad (m \neq \pm n) \qquad \boxed{17}$$