

*I pledge that I have neither given nor received  
unauthorized assistance during this examination.*

**Signature:**

- **DON'T PANIC!** If you get stuck, take a deep breath and go on to the next question.
- Unless the problem says otherwise **you must show your work** sufficiently much that it's clear to me how you arrived at your answer.
- No calculators or electronic devices are allowed.
- Unless the problem specifically says, you **do not** need to calculate the final answer. An answer that looks something like

$$\frac{\binom{6}{2}\binom{8}{5}}{\binom{23}{4}} \quad \text{or} \quad \frac{(12)(11)(10) + (11)(10)(9)}{14!}$$

is entirely acceptable.

- You may bring a two-sided sheet of notes on letter-sized paper in your own handwriting.
- There are 8 problems on 9 pages.

| Question | Points | Score |
|----------|--------|-------|
| 1        | 20     |       |
| 2        | 16     |       |
| 3        | 8      |       |
| 4        | 12     |       |
| 5        | 12     |       |
| 6        | 12     |       |
| 7        | 8      |       |
| 8        | 12     |       |
| Total:   | 100    |       |

**Good luck!**

[20 points] 1. When somebody sends you a text message, you do one of the following, independent of all previous text messages:

- completely ignore the message with probability  $1/6$ ;
- read the message but don't respond with probability  $1/3$ ;
- respond to the message with probability  $1/2$ .

**In the following answers, you don't need to compute a final answer, and you may leave an expression like  $\binom{13}{6}$  in that form. But you should not leave your answer as an infinite sum.**

- (a) What is the probability that you completely ignore the next 10 texts?
- (b) What is the probability that you completely ignore exactly 7 out of the next 10 texts?
- (c) What is the expected number of completely ignored texts out of the next 10?

- (d) What is the probability that out of the first 10 texts, you completely ignore 3, read but don't respond to 2, and respond to 5?
- (e) Number the next text message 1, the next one after that 2, and so on. Let  $X$  be the number of the first text message that you completely ignore. Find  $P(X < 5)$ .

[16 points] 2. Let  $X$  and  $Y$  have joint density

$$f(x, y) = \begin{cases} x + y & \text{if } 0 < x < 1 \text{ and } 0 < y < 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the marginal probability density function of  $X$ . **Fully compute all integrals. Be sure to clearly state where the pdf is zero and where it's nonzero.**

(b) Find  $E(e^{XY})$ . **Set up integrals but do not compute them.**

(c) Find  $P(Y < X)$ . **Set up integrals but do not compute them.**

(d) Find  $P(Y < X^2)$ . **Set up integrals but do not compute them.**

- [8 points] 3. Let  $X \sim \text{Unif}[0, 1]$ , and let  $Y = X^3$ . Compute the probability density function of  $Y$ .  
**Be sure to clearly state where the pdf is zero and where it's nonzero.**



- [12 points] 5. Let  $X$  and  $Y$  be discrete random variables with joint probability mass function given by the following table:

|     |   | $Y$ |    |    |
|-----|---|-----|----|----|
|     |   | 1   | 2  | 3  |
| $X$ | 1 | .1  | .2 | 0  |
|     | 2 | .3  | .1 | .1 |
|     | 3 | 0   | .1 | .1 |

- (a) Give the marginal probability mass functions of  $X$  and  $Y$ .
- (b) Are  $X$  and  $Y$  independent? Justify your answer. (No credit will be given without justification.)
- (c) What is  $P(X = 2 \mid Y = 3)$ ?

- [12 points] 6. A company's revenue on different days form a collection of independent and identically distributed random variables with mean \$1000 and variance 900.
- (a) Let  $X$  be the company's revenue tomorrow. From the information given to you, give the best available bound on the probability that  $X$  is bigger than \$1060.
- (b) Let  $Y$  be the company's revenue over the next 10 days. Use the central limit theorem to give an estimate on the probability that  $Y$  is smaller than \$9900. Write your answer in terms of  $\Phi(x)$ , the cumulative distribution function of the standard normal distribution.



- [8 points] 7. A company has 5 projects to complete and 7 employees. Each employee is assigned to one of the 5 projects, uniformly at random and independently of each other.
- Let  $X$  be the number of projects to which somebody has been assigned. Find  $E(X)$ .

[12 points] 8. Let  $X \sim \text{Exp}(2)$ . That is,  $X$  has density

$$f_X(x) = \begin{cases} 2e^{-2x} & \text{if } x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Let  $Y$  have density

$$f_Y(y) = \begin{cases} 2y & \text{if } 0 < y < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Assume that  $X$  and  $Y$  are independent.

(a) Find  $E(Y)$ .

(b) What is the joint density of  $X$  and  $Y$ ? **Make sure you clearly state where the joint density is equal to zero.**

(c) Find  $E(XY)$ .