Name: ______________________________

On this quiz, you **do not** need to calculate the final answer. An answer that looks something like

$$\frac{\binom{6}{2}\binom{8}{5}}{\binom{23}{4}} \quad \text{or} \quad \frac{(12)_3 + (11)(10)(9)}{14!}$$

is entirely acceptable. **You should compute any integrals, though.**

1. Let Y be a random variable with probability density function $f(x) = \frac{2}{3}(x-1)$ for $x \in [2,3]$. Compute $E\left[(Y-1)^2\right]$.

Solution: Here $g(x) = (x-1)^2$, and we're trying to find $E[g(Y)]$. We use the formula

$$E[g(Y)] = \int_{-\infty}^{\infty} g(x)f(x)\,dx = \int_2^3 (x-1)^2 \cdot \frac{2}{3}(x-1)\,dx$$

The rest of the problem is just working out this integral. You can either multiply out $(x-1)^3$ and then integrate as usual, or you can do a substitution $u = x-1$ which lets you compute the integral as

$$\frac{2}{3}\int_1^2 u^3\,du = \frac{2}{3}\left(\frac{u^4}{4}\right)\Bigg|_1^2 = \frac{2}{3}\left(\frac{16-1}{4}\right) = \frac{5}{2}.$$

There is another question on the back!

2. Let

$$X = \begin{cases} 0 & \text{with probability } 1/3, \\ 1 & \text{with probability } 1/3, \\ 2 & \text{with probability } 1/3. \end{cases}$$

What is $\mathrm{Var}(X)$? (Show enough work that I can follow your calculation.)

Solution: First, we need to find $E(X)$, which in this case is just

$$E(X) = \frac{1}{3}(0+1+2) = 1.$$

Now we find

$$E\big[(X-1)^2\big] = \frac{1}{3}\big((0-1)^2 + (1-1)^2 + (2-1)^2\big) = \frac{2}{3}.$$