Mathematical writing

Algebra skills:

① Transforming expressions

Ex. $3(x+1)^2 = 3(x^2 + 2x + 1)$
$$= 3x^2 + 6x + 3$$

- makes a chain of equalities
- true for all choices of x

- not true for all choices of x — challenge is to figure out which they're true for.

② Solving equations

Solve:
$$2x + 1 = 5x - 4$$
$$1 = 3x - 4$$
$$5 = 3x$$
$$\frac{5}{3} = x$$

- not a chain of equalities, but several different ones
- transforming equations, not expressions, by doing same thing to both sides

Ex. Find the derivative of $(3x+1)(x+2)$.

(transforming an expression)

Solution.

$$\frac{d}{dx}\Big((3x+1)(x+2)\Big) = \frac{d}{dx}\Big(3x^2 + x + 6x + 2\Big)$$

$$= \frac{d}{dx}\Big(3x^2 + 7x + 2\Big)$$

$$= 6x + 7$$

Badly written solution:

$(3x+1)(x+2)$ $3x^2 + x + 6x + 2$ $3x^2 + 7x + 2$ $\boxed{6x+7}$

Ex. (solving an equation)
Find all critical points (where derivative is 0 or undefined) of $f(x) = xe^{-x^2}$

Solution.

$$f'(x) = e^{-x^2} + xe^{-x^2}(-2x) = e^{-x^2} - 2x^2 e^{-x^2} = e^{-x^2}(1 - 2x^2)$$

so far, just transforming an expression

Solve $e^{-x^2}(1 - 2x^2) = 0$

Solutions are solutions to $e^{-x^2} = 0$ or $1 - 2x^2 = 0$

$e^{-x^2} = 0$ has no solutions.

Solve $1 - 2x^2 = 0$:

$1 = 2x^2$

$\frac{1}{2} = x^2$

$x = \pm \frac{1}{\sqrt{2}}$

Don't: cross out or write on top of equations

$\frac{1}{2} = \frac{2x^2}{2}$

$\cancel{3x} + 7 - \cancel{3x} = 2$