Extra Problems

① Take region bounded by $y = \frac{x^2}{10}$, $y = 2$, and the y-axis. Rotate it around y-axis to form a solid.

Set up two integrals to find the volume of this solid using discs and using shells.

② Find $\int \frac{6(\ln x)^4}{x}\,dx$.

③ Find $\int (5x+4)^{1/4}\,dx$.

Chat with me if you have questions and I'll talk one-on-one with you.

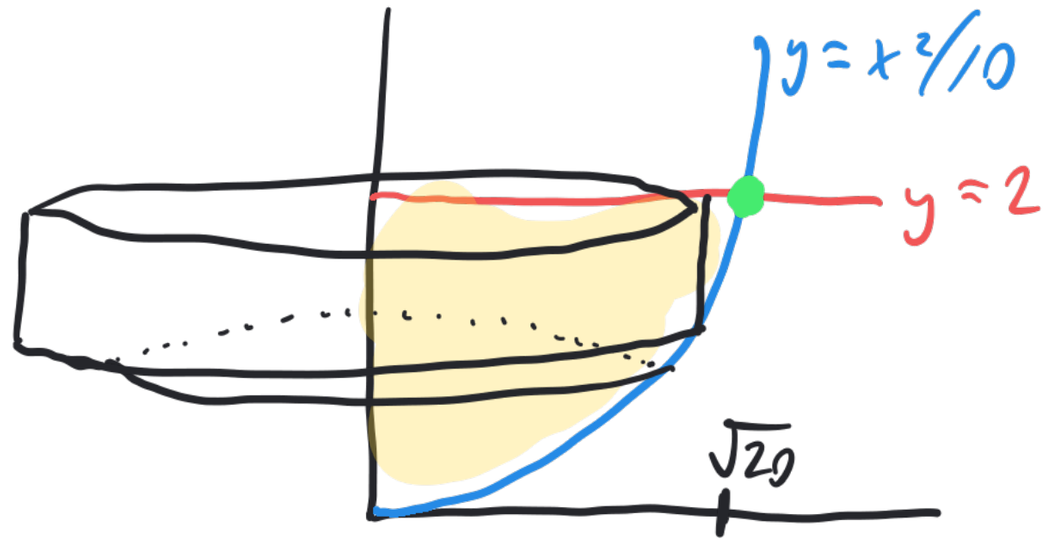
Otherwise, work on problems (here and from exam archive) and we'll go over some later

$y = x^2/10$, or $x = \sqrt{10y}$

$y = 2$

y-axis

Region:



Discs: moving from $y=0$ to $y=2$, discs whose radius is x-coordinate of blue curve.

$$\int_0^2 \pi(\sqrt{10y})^2\,dy = \int_0^2 \pi(10y)\,dy$$

Shells: find x-coordinate of intersection point

$\frac{x^2}{10} = 2$, $x = \sqrt{20}$. Shells have radius x, height $2 - \frac{x^2}{10}$

$$\int_0^{\sqrt{20}} 2\pi x\left(2 - \frac{x^2}{10}\right)dx$$

② $\int \frac{6(\ln x)^4}{x}\,dx$ $\quad u = \ln x \quad du = \frac{1}{x}\,dx$

$= \int 6u^4\,du = \frac{6u^5}{5} + C = \frac{6(\ln x)^5}{5} + C$

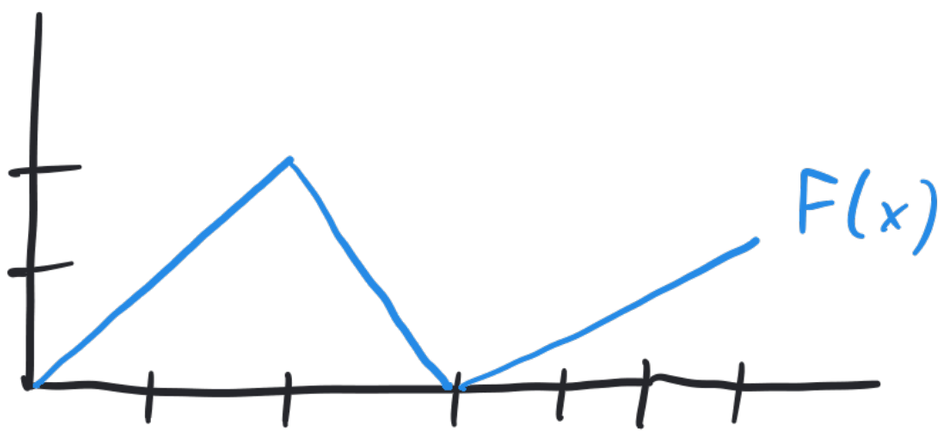
③ $\int (5x+4)^{1/4}\,dx$ $\quad u = 5x+4 \quad du = 5\,dx$

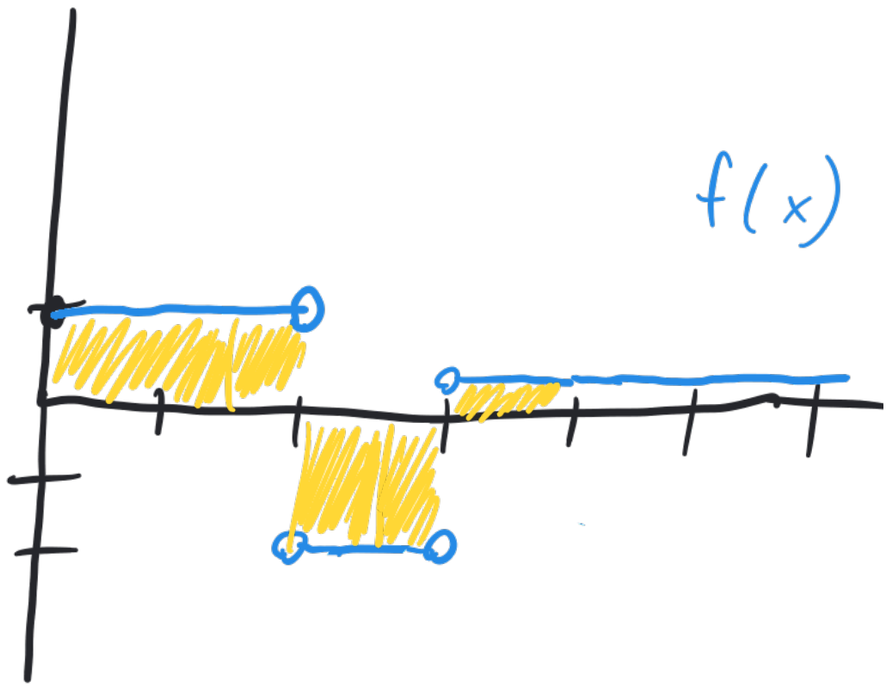
$\frac{1}{5}\,du = dx$

$= \int u^{1/4} \cdot \frac{1}{5}\,du$

$= \frac{1}{5}\,\frac{u^{5/4}}{5/4} = \frac{4u^{5/4}}{25} + C = \frac{4(5x+4)^{5/4}}{25} + C$

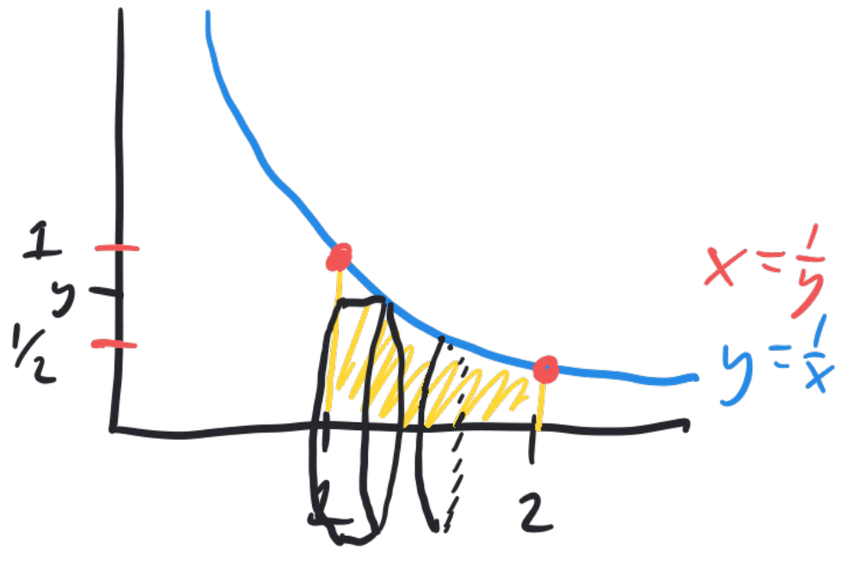
$$F(x) = \int_0^x f(t)\,dt$$





#3 Consider region under $f(x) = \frac{1}{x}$ from $x = 1$ to $x = 2$

Rotate around x-axis. Find volume



Discs: integrate from 1 to 2 w.r.t. x,
discs have radius $\frac{1}{x}$.

$$\int_1^2 \pi \left(\frac{1}{x}\right)^2 dx$$

Shells: integrate from 0 to 1 w.r.t. y

Shells at given y has radius y. For $y \leq \frac{1}{2}$, height of shell is $2 - 1 = 1$.

For $\frac{1}{2} \leq y \leq 1$, height of shell is $\frac{1}{y} - 1$.

$$\int_0^{1/2} 2\pi y(1)\,dy + \int_{1/2}^1 2\pi y\left(\frac{1}{y} - 1\right) dy$$