Name: ______________________________

Note: When transforming expressions into equivalent forms, write a chain of equalities. Don't just write one expression after the next without stating the relationship between the expressions.

When solving equations, write down one equation after another. Do not cross out or write on top of an equation.

Do the following series converge or diverge? Justify your answers.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^9}$

Solution: The limiting ratio is

$$\rho = \lim_{n\to\infty} \frac{2^{n+1}/(n+1)^9}{2^n/n^9} = \lim_{n\to\infty} 2\left(\frac{n}{n+1}\right)^9 = \lim_{n\to\infty} 2\left(\frac{1}{1+\frac{1}{n}}\right)^9 = \lim_{n\to\infty} 2\left(\frac{1}{1+0}\right)^9 = 2.$$

Since $\rho > 1$, the series diverges by the ratio test.

2. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+1}{n^2+1}\right)^n$

Solution: Apply the root test:

$$L = \lim_{n\to\infty} \left(\left(\frac{n+1}{n^2+1}\right)^n\right)^{1/n} = \lim_{n\to\infty} \left(\frac{n+1}{n^2+1}\right) = \lim_{n\to\infty} \left(\frac{1+\frac{1}{n}}{n+\frac{1}{n}}\right) = 0.$$

So the series converges.

3. $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n!}$

Solution: Apply the ratio test:

$$\rho = \lim_{n\to\infty} \frac{\frac{n+2}{(n+1)!}}{\frac{n+1}{n!}} = \lim_{n\to\infty} \left(\frac{n+2}{n+1}\right)\left(\frac{n!}{(n+1)!}\right) = \lim_{n\to\infty} \left(\frac{1+\frac{2}{n}}{1+\frac{1}{n}}\right)\left(\frac{1}{n+1}\right) = 0.$$

So the series converges.