Name: ______________________________

Note: When transforming expressions into equivalent forms, write a chain of equalities. Don't just write one expression after the next without stating the relationship between the expressions.

When solving equations, write down one equation after another. Do not cross out or write on top of an equation.

1. Find $\displaystyle\int \frac{3(x^2-x+1)}{(x+1)(x-2)^2}\,dx$.

Solution: Set

$$\frac{3(x^2-x+1)}{(x+1)(x-2)^2} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

and multiply both sides by $(x+1)(x-2)^2$ to get

$$3(x^2-x+1) = A(x-2)^2 + B(x+1)(x-2) + C(x+1).$$

Set $x=-1$ to get

$$9 = 9A,$$

giving $A=1$. Set $x=2$ to get

$$9 = 3C,$$

giving $C=3$. Now substitute $x=0$ (or anything else will work, too) to get

$$3 = 4A - 2B + C = 4 - 2B + 3$$

to get $B=2$.

Now, we integrate:

$$\begin{aligned}\frac{3(x^2-x+1)}{(x+1)(x-2)^2}\,dx &= \int\left(\frac{1}{x+1} + \frac{2}{x-2} + \frac{3}{(x-2)^2}\right)dx \\ &= \ln|x+1| + 2\ln|x-2| - \frac{3}{x-2} + C.\end{aligned}$$

2. Find $\int x^3 \ln x \, dx$.

Solution: Do integration by parts with $u = \ln x$ and $dv = x^3 \, dx$. Then $du = \frac{1}{x} \, dx$ and $v = \frac{x^4}{4}$, and

$$\begin{aligned} \int x^3 \ln x \, dx &= (\ln x)\frac{x^4}{4} - \int \left(\frac{x^4}{4}\right)\left(\frac{1}{x}\right) dx \\ &= \frac{x^4 \ln x}{4} - \frac{1}{4}\int x^3 \, dx = \frac{x^4 \ln x}{4} - \frac{x^4}{16} + C. \end{aligned}$$