Name: ______________________________

Note: When transforming expressions into equivalent forms, write a chain of equalities. Don't just write one expression after the next without stating the relationship between the expressions.

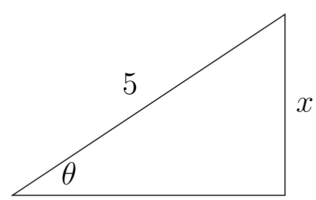
When solving equations, write down one equation after another. Do not cross out or write on top of an equation.

1. Find $\displaystyle\int \frac{x^2}{\sqrt{25-x^2}}\,dx$.

Solution: Substitute $x = 5\sin\theta$. Then $dx = 5\cos\theta\,d\theta$, and the integral becomes

$$\int \frac{25\sin^2\theta}{\sqrt{25-25\sin^2\theta}}\,5\cos\theta\,d\theta = \int \frac{25\sin^2\theta}{5\cos\theta}\,5\cos\theta\,d\theta = 25\int \sin^2\theta\,d\theta = \frac{25}{2}(\theta - \sin\theta\cos\theta) + C.$$

Now we have to substitute back for x. Since $x = 5\sin\theta$, we have $\theta = \sin^{-1}(x/5)$. Also $\sin\theta = x/5$, but to finish back-substituting we need to work out $\cos\theta$. We make a right triangle with an angle θ whose sine is $x/5$, like this:



The other side has length $\sqrt{25-x^2}$ by the Pythagorean theorem. So $\cos\theta = \sqrt{25-x^2}/5$. And now the integral is

$$\frac{25}{2}\left(\sin^{-1}(x/5) - \frac{x}{5}\frac{\sqrt{25-x^2}}{5}\right) + C = \frac{25}{2}\sin^{-1}(x/5) - \frac{x\sqrt{25-x^2}}{2} + C$$

TABLE OF TRIGONOMETRIC INTEGRALS

$$\int \sin^2 x\, dx = \frac{x}{2} - \frac{\sin 2x}{4} + C = \frac{x}{2} - \frac{1}{2}\sin x \cos x + C \tag{3}$$

$$\int \cos^2 x\, dx = \frac{x}{2} + \frac{\sin 2x}{4} + C = \frac{x}{2} + \frac{1}{2}\sin x \cos x + C \tag{4}$$

$$\int \sin^n x\, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\, dx \tag{5}$$

$$\int \cos^n x\, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\, dx \tag{6}$$

$$\int \tan x\, dx = \ln|\sec x| + C = -\ln|\cos x| + C \tag{7}$$

$$\int \tan^m x\, dx = \frac{\tan^{m-1} x}{m-1} - \int \tan^{m-2} x\, dx \tag{8}$$

$$\int \cot x\, dx = -\ln|\csc x| + C = \ln|\sin x| + C \tag{9}$$

$$\int \cot^m x\, dx = -\frac{\cot^{m-1} x}{m-1} - \int \cot^{m-2} x\, dx \tag{10}$$

$$\int \sec x\, dx = \ln\left|\sec x + \tan x\right| + C \tag{11}$$

$$\int \sec^m x\, dx = \frac{\tan x \sec^{m-2} x}{m-1} + \frac{m-2}{m-1}\int \sec^{m-2} x\, dx \tag{12}$$

$$\int \csc x\, dx = \ln\left|\csc x - \cot x\right| + C \tag{13}$$

$$\int \csc^m x\, dx = -\frac{\cot x \csc^{m-2} x}{m-1} + \frac{m-2}{m-1}\int \csc^{m-2} x\, dx \tag{14}$$

$$\int \sin mx \sin nx\, dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + C \quad (m \neq \pm n) \tag{15}$$

$$\int \sin mx \cos nx\, dx = -\frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)} + C \quad (m \neq \pm n) \tag{16}$$

$$\int \cos mx \cos nx\, dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + C \quad (m \neq \pm n) \tag{17}$$