Math 232, Midterm 2, Version A Name: ______________________

March 30, 2026

I pledge that I have neither given nor received unauthorized assistance during this examination.

Signature:

- **DON'T PANIC!** If you get stuck, take a deep breath and go on to the next question.
- Unless the problem says otherwise **you must show your work** sufficiently much that it's clear to me how you arrived at your answer.
- When transforming expressions into equivalent forms, write a chain of equalities. Don't just write one expression after the next without stating the relationship between the expressions.
- When solving equations, write down one equation after another. Do not cross out or write on top of an equation.
- You can use a calculator, graphing or otherwise. You cannot use a computer or phone.
- You may bring a two-sided sheet of notes on letter-sized paper in your own handwriting.
- There are 7 problems on 7 pages.

Question	Points	Score
1	8	
2	6	
3	8	
4	8	
5	9	
6	6	
7	8	
Total:	53	

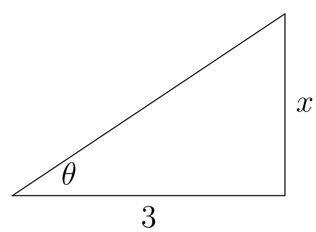
Good luck!

[8 points] 1. Compute $\displaystyle\int \frac{1}{(9+x^2)^{3/2}}\,dx$.

Solution: Make the substitution $x = 3\tan\theta$. Then $dx = 3\sec^2\theta\,d\theta$, and the substitution gives

$$\begin{aligned}\int \frac{1}{(9+x^2)^{3/2}}\,dx = \int \frac{3\sec^2\theta}{(9+9\tan^2\theta)^{3/2}}\,d\theta &= \int \frac{3\sec^2\theta}{(9\sec^2\theta)^{3/2}}\,d\theta \\ &= \int \frac{3\sec^2\theta}{27\sec^3\theta}\,d\theta \\ &= \frac{1}{9}\int \cos\theta\,d\theta = \frac{1}{9}\sin\theta + C.\end{aligned}$$

Now, we have to work out $\sin\theta$ in terms of x. We know that $\tan\theta = x/3$. So we make a right triangle with an angle θ whose tangent is $x/3$, like this:



The other side has length $\sqrt{9+x^2}$ by the Pythagorean theorem. And this makes $\sin\theta = x/\sqrt{9+x^2}$. So,

$$\int \frac{1}{(9+x^2)^{3/2}}\,dx = \frac{x}{9\sqrt{9+x^2}} + C.$$

[6 points] 2. Compute $\displaystyle\int \frac{2x+5}{x^2+2x}\,dx$.

Solution: Factor x^2+2x as $x(x+2)$ and set

$$\frac{2x+5}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}.$$

Now multiply both sides by $x(x+2)$ to get

$$2x+5 = A(x+2) + Bx.$$

Set $x=-2$ to get $1=-2B$, giving $B=-1/2$. And set $x=0$ to get $5=2A$, giving $A=\frac{5}{2}$. So,

$$\int \frac{2x+5}{x^2+2x}\,dx = \int \left(\frac{5/2}{x} - \frac{1/2}{x+2}\right) dx = \frac{5}{2}\ln|x| - \frac{1}{2}\ln|x+2| + C.$$

[8 points] 3. Compute the following series or state that they do not converge. Explain your answer. If you apply a test, you must give all details of the test to get full credit. (For example, for the comparison test, say what series you're comparing to.)

(a) $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^2$

Solution: The series diverges. One way to see this is that $\left(1+\frac{1}{n}\right)^2 \geq 1$, and the series $\sum_{n=1}^{\infty} 1$ diverges. Another way is to observe that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^2 = 1$, which is not equal to zero, showing that the series diverges.

(b) $\displaystyle\sum_{n=0}^{\infty}\frac{3^n}{5^{n+1}}$

Solution: This is a geometric series with first term $1/5$ and multiplier $3/5$. So it converges to $\frac{1/5}{1-3/5} = \frac{1}{2}$.

[8 points] 4. Compute the following improper integrals (or state that they diverge).

(a) $\int_0^\infty e^{-3x}\,dx$

Solution: First,

$$\int_0^R e^{-3x}\,dx = -\tfrac{1}{3}e^{-3x}\Big|_0^R = -\tfrac{1}{3}e^{-3R} + \tfrac{1}{3}.$$

Now,

$$\int_0^\infty e^{-3x}\,dx = \lim_{R\to\infty}\int_0^R e^{-3x}\,dx = \lim_{R\to\infty}\left(-\tfrac{1}{3}e^{-3R} + \tfrac{1}{3}\right) = \tfrac{1}{3},$$

since $e^{-3R} \to 0$ as $R \to \infty$.

(b) $\int_0^\infty \frac{x}{(x^2+3)^3}\,dx$

Solution: We start by making the substitution $u = x^2 + 3$, which makes $du = 2x\,dx$. The integral transforms to

$$\frac{1}{2}\int_3^\infty \frac{du}{u^3}.$$

Now we compute

$$\frac{1}{2}\int_3^R \frac{du}{u^3} = \frac{1}{2}\left(-\frac{1}{2u^2}\right)\Bigg|_3^R = -\frac{1}{4R^2} + \frac{1}{36}.$$

Now

$$\int_0^\infty \frac{x}{(x^2+3)^3}\,dx = \frac{1}{2}\int_3^\infty \frac{du}{u^3} = \lim_{R\to\infty}\left(-\frac{1}{4R^2} + \frac{1}{36}\right) = \frac{1}{36}.$$

[9 points] 5. Compute the following integrals:

(a) $\int xe^{3x}\,dx$

Solution: Do integration by parts with $u = x$ and $dv = e^{3x}\,dx$. Then $du = dx$ and $v = \frac{1}{3}e^{3x}$, and

$$\int xe^{3x}\,dx = \frac{xe^{3x}}{3} - \frac{1}{3}\int e^{3x}\,dx = \frac{xe^{3x}}{3} - \frac{e^{3x}}{9} + C$$

(b) $\int x\ln x\,dx$

Solution: Do integration by parts with $u = \ln x$ and $dv = x\,dx$. Then $du = \frac{1}{x}\,dx$ and $v = x^2/2$, and

$$\begin{aligned}\int x\ln x\,dx &= \frac{x^2\ln x}{2} - \frac{1}{2}\int x^2\left(\frac{1}{x}\right)dx \\ &= \frac{x^2\ln x}{2} - \frac{1}{2}\int x\,dx = \frac{x^2\ln x}{2} - \frac{x^2}{4} + C.\end{aligned}$$

(c) $\int_0^2 \frac{1}{\sqrt{x+1}}\,dx$

Solution: Substitute $u = x + 1$ to get

$$\int_0^2 \frac{1}{\sqrt{x+1}}\,dx = \int_1^3 \frac{1}{\sqrt{u}}\,du = 2\sqrt{u}\Big|_1^3 = 2\sqrt{3} - 2.$$

[6 points] 6. Find the partial fractions decomposition of $\dfrac{17-x^2}{(x+1)(x-3)^2}$. (You do not have to integrate it.)

Solution: Set

$$\frac{17-x^2}{(x+1)(x-3)^2} = \frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

and multiply both sides by $(x+1)(x-3)^2$ to get

$$17-x^2 = A(x-3)^2 + B(x+1)(x-3) + C(x+1)$$

Substitute $x=3$ to get $8=4C$, yielding $C=2$. Substitute $x=-1$ to get $16=16A$, yielding $A=1$. Now substitute $x=0$ (or anything else) to get

$$17 = 9A - 3B + C = 9 - 3B + 2,$$

yielding $B=-2$. So the decomposition is

$$\frac{17-x^2}{(x+1)(x-3)^2} = \frac{1}{x+1} - \frac{2}{x-3} + \frac{2}{(x-3)^2}.$$

[8 points] 7. State whether the following series converge or diverge. Explain your answer. If you apply a test, you must give all details of the test to get full credit. (For example, for the comparison test, say what series you're comparing to.)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2+2n+2}$

Solution: There are different ways to approach this problem, but the easiest is probably to compare the series to $\sum_{n=1}^{\infty} \frac{1}{n}$ via the use the limit comparison test. We compute

$$\lim_{n\to\infty} \frac{\frac{n+1}{n^2+2n+2}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^2+n}{n^2+2n+2} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{1+\frac{2}{n}+\frac{2}{n^2}} = 1.$$

Since this limit is equal to 1, the series $\sum_{n=1}^{\infty} \frac{n+1}{n^2+2n+2}$ converges if and only if $\sum_{n=1}^{\infty} \frac{1}{n}$ does. And we know that this second series does not converge.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3 10^n}$

Solution: Since $\frac{1}{n^3 10^n} \leq \frac{1}{10^n}$ for all $n \geq 1$, and the series $\sum_{n=1}^{\infty} \frac{1}{10^n}$ converges (since it's geometric with multiplier $1/10$), this series converges too by the comparison test.