The formulas in Eqs. (15)–(17) in the integral table at the end of the section give the integrals of the products $\sin mx \sin nx$, $\cos mx \cos nx$, and $\sin mx \cos nx$. These integrals appear in the theory of Fourier Series, which is a fundamental technique used extensively in engineering and physics.

EXAMPLE 10 Integral of $\sin mx \cos nx$ Evaluate $\displaystyle\int_0^{\pi} \sin 4x \cos 3x\, dx$.

Solution Apply Eq. (16), with $m = 4$ and $n = 3$:

$$\begin{aligned}\int_0^{\pi} \sin 4x \cos 3x\, dx &= \left(-\frac{\cos(4-3)x}{2(4-3)} - \frac{\cos(4+3)x}{2(4+3)}\right)\Bigg|_0^{\pi} \\ &= \left(-\frac{\cos x}{2} - \frac{\cos 7x}{14}\right)\Bigg|_0^{\pi} \\ &= \left(\frac{1}{2} + \frac{1}{14}\right) - \left(-\frac{1}{2} - \frac{1}{14}\right) = \frac{8}{7}\end{aligned}$$

■

The following table of trigonometric integrals summarizes some of the integral formulas we have seen in this chapter and includes some other related formulas.

TABLE OF TRIGONOMETRIC INTEGRALS

$$\int \sin^2 x\, dx = \frac{x}{2} - \frac{\sin 2x}{4} + C = \frac{x}{2} - \frac{1}{2}\sin x \cos x + C \qquad 3$$

$$\int \cos^2 x\, dx = \frac{x}{2} + \frac{\sin 2x}{4} + C = \frac{x}{2} + \frac{1}{2}\sin x \cos x + C \qquad 4$$

$$\int \sin^n x\, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\, dx \qquad 5$$

$$\int \cos^n x\, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\, dx \qquad 6$$

$$\int \tan x\, dx = \ln|\sec x| + C = -\ln|\cos x| + C \qquad 7$$

$$\int \tan^m x\, dx = \frac{\tan^{m-1} x}{m-1} - \int \tan^{m-2} x\, dx \qquad 8$$

$$\int \cot x\, dx = -\ln|\csc x| + C = \ln|\sin x| + C \qquad 9$$

$$\int \cot^m x\, dx = -\frac{\cot^{m-1} x}{m-1} - \int \cot^{m-2} x\, dx \qquad 10$$

$$\int \sec x\, dx = \ln|\sec x + \tan x| + C \qquad 11$$

$$\int \sec^m x\, dx = \frac{\tan x \sec^{m-2} x}{m-1} + \frac{m-2}{m-1}\int \sec^{m-2} x\, dx \qquad 12$$

$$\int \csc x\, dx = \ln|\csc x - \cot x| + C \qquad 13$$

$$\int \csc^m x\, dx = -\frac{\cot x \csc^{m-2} x}{m-1} + \frac{m-2}{m-1}\int \csc^{m-2} x\, dx \qquad 14$$

$$\int \sin mx \sin nx\, dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + C \quad (m \neq \pm n) \qquad 15$$

$$\int \sin mx \cos nx\, dx = -\frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)} + C \quad (m \neq \pm n) \qquad 16$$

$$\int \cos mx \cos nx\, dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + C \quad (m \neq \pm n) \qquad 17$$