Name: ______________________________

1. Compute the derivatives of the following functions. You do not need to algebraically simplify your answers. For example, if $f(x) = 10x^3$, I would rather that you wrote $f'(x) = 10(3)x^2$ than $f'(x) = 30x^2$ (it's easier to grade!).

   (a) $f(x) = \sqrt{1 - x^3}$

   Solution:

   $$f'(x) = \frac{1}{2\sqrt{1-x^3}}\left(-3x^2\right)$$

   (b) $g(t) = te^{t^2}$

   Solution:

   $$g'(t) = e^{t^2} + te^{t^2}(2t)$$

   (c) $h(x) = \Big(\sin(2x+1)\Big)^3$

   Solution:

   $$h'(x) = 3\Big(\sin(2x+1)\Big)^2 \cos(2x+1)(2)$$

2. A large hot air balloon begins its descent by releasing air at a rate of 100 cubic meters per minute. The balloon is a sphere. What is the rate of change of the radius when it is 10 meters? (The formula for the volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

Solution: Let r be the radius and V the volume of the balloon. We know that $\frac{dV}{dt} = -100 \text{ m}^3/\text{min}$. Our goal is to find $\frac{dr}{dt}\big|_{r=10}$. We use the volume formula to relate r and V. Taking derivatives of both sides gives

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}.$$

We substitute $\frac{dV}{dt} = -100$ and $r = 10$ and get

$$-100 = 4\pi\left(10^2\right)\frac{dr}{dt}.$$

Solving yields $\frac{dr}{dt} = -\frac{1}{4\pi} \approx -0.080$ m/min.