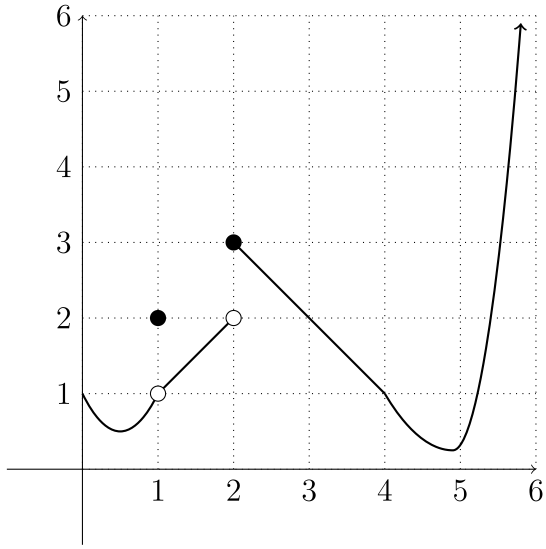
Name: ______________________________

1. Here is the graph of a function $f(x)$. Note that there is a vertical asymptote at $x = 6$.



Find the following quantities. If a limit does not exist, say so. (If a limit diverges to infinity, it is acceptable either to say that it doesn't exist, or to say that it diverges to infinity, or to write down that its limit is ∞ or $-\infty$.)

(a) $\lim_{x\to 1^-} f(x) =$

Solution: 1

(b) $\lim_{x\to 1^+} f(x) =$

Solution: 1

(c) $\lim_{x\to 1} f(x) =$

Solution: 1

(d) $f(1) =$

Solution: 2

(e) $\lim_{x\to 2^-} f(x) =$

Solution: 2

(f) $\lim_{x \to 2^+} f(x) =$

Solution: 3

(g) $\lim_{x \to 2} f(x) =$

Solution: doesn't exist

(h) $f(2) =$

Solution: 3

(i) $\lim_{x \to 3} f(x) =$

Solution: 2

(j) $\lim_{x \to 6^-} f(x) =$

Solution: ∞, or say that the limit diverges to infinity, or say that the limit doesn't exist

2. Find $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$.

Solution: Simplify the fraction as

$$\frac{x^2 - 25}{x - 5} = \frac{(x-5)(x+5)}{x-5} = x + 5.$$

(This doesn't work when $x = 5$, but that doesn't affect the limit.) Now

$$\lim_{x \to 5} \frac{x^2 - 25}{x - 5} = \lim_{x \to 5}(x+5) = 5 + 5 = 10.$$

3. Expand the algebraic expression $x(2x+3)^2$ into a cubic polynomial, i.e., a polynomial of the form $ax^3 + bx^2 + cx + d$ for constants a, b, c, d. **Please note:**

- You are expanding an algebraic expression, so your solution should look like a chain of equalities. That is, it should begin $x(2x+3)^2 = \ldots$ and then be followed by more equalities until you arrive at a cubic polynomial.
- Write down this chain of equalities one after the other on the page, not scattered all around.

- Do not cross out or write on top of any equation; do not draw arrows that don't have any mathematical meaning.

Solution:

$$x(2x+3)^2 = x(4x^2+12x+9) = 4x^3+12x^2+9x.$$

In the first step, we're using the fact $(a+b)^2 = a^2+2ab+b^2$. Then we're distributing the final x.