Basic differentiation

Covered in sections 3.3–3.7, 3.9.

1. Find derivatives of the following functions:
   (a) $f(\theta) = \sin(\ln\theta)$
   (b) $h(t) = t^{(t^t)}$
   (c) $f(x) = x^2 e^{1/x}$

2. Find equations for the tangent line to the graph of f at $x = a$:
   (a) $f(x) = x^2 - x, \quad a = 1$
   (b) $f(x) = 5 - 3x, \quad a = 2$

Limits (including L'Hôpital's rule)

Covered in sections 2.3–2.7 and 4.5.

3. Evaluate the limit or state that it doesn't exist.
   (a)
   $$\lim_{x\to\infty} \frac{x^2 - 3x^4}{x - 1}$$
   (b)
   $$\lim_{x\to 1} x^{1/(x-1)}$$
   (c)
   $$\lim_{x\to 0} \frac{x^3}{\sin x - x}$$
   (d)
   $$\lim_{x\to 3} \frac{2x^2 - 5x - 3}{x - 4}$$

Implicit Differentiation

Covered in section 3.8.

4. Find an equation for the line tangent to the curve $x^2 + \sin y = xy^2 + 1$ at the point $(1, 0)$.

Related Rates

Covered in section 3.10.

5. A road perpendicular to a highway leads to a farmhouse located 2 km off the highway. An automobile travels on the highway at 80 km/h. How fast is the distance between the automobile and farmhouse increasing when the automobile is 6 km past the intersection of the highway and road?

Linear Approximation

Covered in section 4.1.

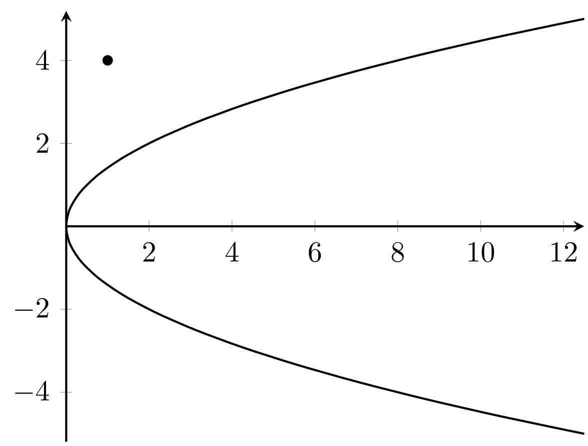
6. Let $P = (2, 1)$, a point on the curve $y^3 + 3xy = 7$. Give the approximate y-coordinate of the point on the curve near P with x-coordinate 2.1.

Maxima, minima, and optimization

Covered in sections 4.2, 4.7.

7. Find the maximum value of $f(x) = 2\sqrt{x} - x$ on $[0, 4]$.

8. Find the point on the curve $y^2 = 2x$ closest to $(1, 4)$.



The shape of a graph

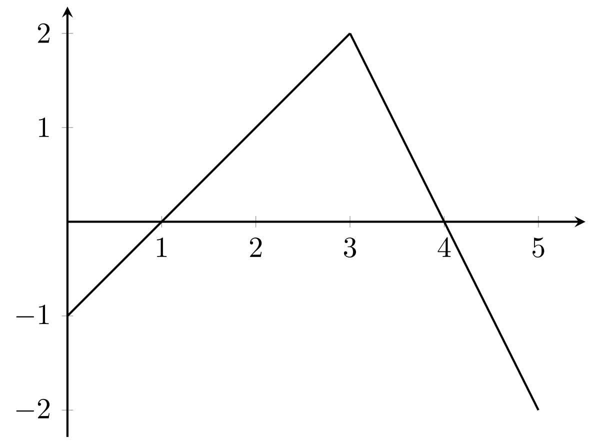
Covered in sections 4.3, 4.4, and 4.6.

9. Sketch the graph of $y = xe^{-x^2}$. Be accurate with regard to whether the graph is increasing or decreasing, its concavity, and its asymptotic behavior.

Area and definite integrals

Covered in sections 5.1, 5.2.

10. The following is a graph of $y = g(x)$.



Evaluate $\displaystyle\int_0^5 g(t)\,dt$.

11. Compute R_5, the right endpoint approximation with 5 rectangles, for the area under the curve $f(x) = x^2 + x$ from -1 to 1.

Antiderivatives, the fundamental theorem of calculus, and integration techniques

Covered in sections 5.3–5.5, 5.7–5.8.

12. Compute the following integrals.

(a)

$$\int (\sqrt{t} + 1)(t + 1)\,dt$$

(b)

$$\int_{-2}^{0} (3x - 9e^{3x})dx$$

(c) Find

$$\frac{d}{dx}\int_1^{x^4} \sqrt{t}\,dt$$

(d)

$$\int_0^1 \frac{x}{(x^2+1)^3}dx$$

(e)

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx$$