Math 231, Midterm 1, version A
September 25, 2019

Name: ____________________

I pledge that I have neither given nor received unauthorized assistance during this examination.

Signature:

- **DON'T PANIC!** If you get stuck, take a deep breath and go on to the next question.
- Unless the problem says otherwise **you must show your work** sufficiently much that it's clear to me how you arrived at your answer.
- You may use a scientific calculator on this exam, but you may not use a graphing calculator.
- You may bring a two-sided sheet of notes on letter-sized paper in your own handwriting.
- There are 6 problems on 9 pages.

Question	Points	Score
1	16	
2	14	
3	12	
4	12	
5	12	
6	11	
Total:	77	

Good luck!

[16 points] 1. (a)

$$\lim_{x \to -2} \frac{x^2 + 6x + 8}{x + 2} =$$

Solution:

$$\lim_{x \to -2} \frac{x^2 + 6x + 8}{x + 2} = \lim_{x \to 2} \frac{(x+2)(x+4)}{x+2} = \lim_{x \to -2} (x+4) = 2.$$

(b)

$$\lim_{y \to 1} \frac{y^2 + y - 5}{-y^3 - 2y - 1} =$$

Solution:

$$\lim_{y \to 1} \frac{y^2 + y - 5}{-y^3 - 2y - 1} = \frac{1^2 + 1 - 5}{-1^3 - 2(1) - 1} = \frac{1 + 1 - 5}{-1 - 2 - 1} = \frac{-3}{-4} = \frac{3}{4}.$$

(c)

$$\lim_{x \to \infty} \frac{\sqrt{4x^4 + x^2 + 3x + 1}}{x^2 + 1} =$$

Solution:

$$\begin{aligned}
\lim_{x \to \infty} \frac{\sqrt{4x^4 + x^2 + 3x + 1}}{x^2 + 1} &= \lim_{x \to \infty} \frac{\sqrt{4x^4 + x^2 + 3x + 1}}{x^2 + 1} \frac{\frac{1}{x^2}}{\frac{1}{x^2}} \\
&= \lim_{x \to \infty} \frac{\sqrt{(4x^4 + x^2 + 3x + 1)\frac{1}{x^4}}}{1 + \frac{1}{x^2}} \\
&= \lim_{x \to \infty} \frac{\sqrt{4 + \frac{1}{x^2} + \frac{3}{x^3} + \frac{1}{x^4}}}{1 + \frac{1}{x^2}} \\
&= \frac{\sqrt{4 + 0 + 0 + 0}}{1 + 0} = 2.
\end{aligned}$$

(d)

$$\lim_{t \to -\infty} \frac{2t^2 - t + 2}{t^3 - 4} =$$

Solution:

$$\lim_{t\to-\infty} \frac{2t^2-t+2}{t^3-4} = \lim_{t\to-\infty} \frac{2t^2-t+2}{t^3-4}\frac{\frac{1}{t^2}}{\frac{1}{t^2}} = \lim_{t\to-\infty} \frac{2-\frac{1}{t}+\frac{2}{t^2}}{t-\frac{4}{t^2}} = 0,$$

since "$\frac{2}{\infty}$" $= 0$.

2. A ball is thrown directly upward in the air. Its height in meters at time t seconds is given by the formula

$$h(t) = -5t^2 + 20t.$$

[6 points] (a) What is the average velocity of the ball from time 0 to time 2?

Solution:

$$\frac{h(2) - h(0)}{2 - 0} = \frac{20 - 0}{2 - 0} = 10 \text{ meters/second.}$$

[8 points] (b) How fast is the ball moving at time 2?

Solution: First, we compute $h'(t)$:

$$h'(t) = -10t + 20.$$

Now the answer is

$$h'(2) = -10(2) + 20 = 0 \text{ meters/second.}$$

[12 points] 3. Let

$$f(x) = \frac{x^2+1}{3x-2}.$$

(a) Does this function have any vertical asymptotes? If so, where do they occur? Justify your answer (briefly).

> **Solution:** There is a vertical asymptote at $x = 2/3$, because $\lim_{x\to 2/3} f(x)$ diverges to infinity there (we can see this because the bottom of the fraction converges to 0 and the top converges to something nonzero.

(b) Does this function have any horizontal asymptotes? If so, what are their values? Justify your answer (briefly).

> **Solution:** Since $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = \infty$, there are no horizontal asymptotes. For there to be a horizontal asymptote, one of these limits would have to converge to a finite number.

[12 points] 4. (a) Compute $\frac{d}{dx}(2x^3 - 7x + 1)$.

Solution:

$$\frac{d}{dx}(2x^3 - 7x + 1) = 6x^2 - 7.$$

(b) Let $g(u) = 3u^4 - \frac{1}{\sqrt{u}}$. Find $g'(1)$.

Solution: Since $g(u) = 3u^4 - u^{-1/2}$.

$$g'(u) = 12u^3 + \frac{1}{2}u^{-3/2}.$$

So,

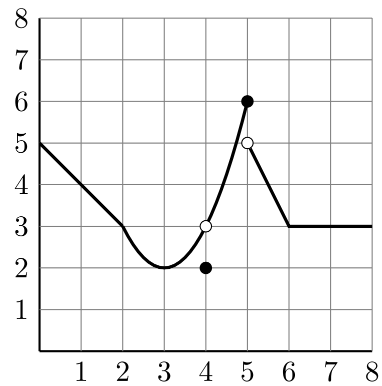
$$g'(1) = 12 + \frac{1}{2} = 12.5.$$

[12 points] 5. Let $f(x) = (x+1)^2$. Using the definition of the derivative directly, compute $f'(2)$. **Warning:** You will not receive credit for using differentiation rules. You must use limits to compute the derivative directly from the definition.

Solution:

$$f'(2) = \lim_{b\to 2} \frac{f(b)-f(2)}{b-2} = \lim_{b\to 7} \frac{(b+1)^2-(2+1)^2}{b-2} = \lim_{b\to 2} \frac{b^2+2b+1-9}{b-2} = \lim_{b\to 2} \frac{b^2+2b-8}{b-2}$$
$$= \lim_{b\to 2} \frac{(b-2)(b+4)}{b-2} = \lim_{b\to 2}(b+4) = 6.$$

[11 points] 6. Here is the graph of a function $f(x)$ on the domain $0 \le x \le 8$:



Evaluate the following expressions. If the value does not exist, say so. You do not need to justify your answers.

(a) $f(1) =$

Solution: 4

(b) $\lim_{x\to 1} f(x) =$

Solution: 4

(c) $f'(1) =$

Solution: -1

(d) $f'(3) =$

Solution: 0

(e) $\lim_{x\to 4} f(x) =$

Solution: 3

(f) $f(5) =$

Solution: 6

(g) $\lim_{x\to 5^+} f(x) =$

Solution: 5

(h) $\lim_{x\to 5} f(x) =$

Solution: doesn't exist

(i) $\lim_{x\to 6} f(x) =$

Solution: 3

(j) $f'(6) =$

Solution: doesn't exist

(k) $f'(7) =$

Solution: 0

This page can be used as scratch paper.