Math 231, Midterm 1, version A Name: ______________________________

March 1, 2018

I pledge that I have neither given nor received unauthorized assistance during this examination.

Signature:

- **DON'T PANIC!** If you get stuck, take a deep breath and go on to the next question.
- Unless the problem says otherwise **you must show your work** sufficiently much that it's clear to me how you arrived at your answer.
- You may use a scientific calculator on this exam, but you may not use a graphing calculator.
- You may bring a two-sided sheet of notes on letter-sized paper in your own handwriting.
- There are 6 problems on 7 pages.

Question	Points	Score
1	16	
2	18	
3	16	
4	10	
5	14	
6	9	
Total:	83	

Good luck!

[16 points] 1. (a)

$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9} =$$

Solution:

$$\begin{aligned}\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9} &= \lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right)\\ &= \lim_{x\to 9}\frac{x-9}{(x-9)(\sqrt{x}+3)} = \lim_{x\to 9}\frac{1}{\sqrt{x}+3} = \frac{1}{6}.\end{aligned}$$

(b)

$$\lim_{y\to\infty}\frac{y^2+y-5}{1-2y-y^3} =$$

Solution: The top and the bottom of the fraction converge to $+\infty$ and $-\infty$, which is indeterminate and tells us nothing. We manipulate the expression like this:

$$\begin{aligned}\lim_{y\to\infty}\frac{y^2+y-5}{1-2y-y^3} &= \lim_{y\to\infty}\frac{y^2+y-5}{1-2y-y^3}\left(\frac{\frac{1}{y^2}}{\frac{1}{y^2}}\right)\\ &= \lim_{y\to\infty}\frac{1+\frac{1}{y}-\frac{5}{y^2}}{\frac{1}{y^2}-\frac{2}{y}-y}\end{aligned}$$

The top of this fraction converges to 1 and the bottom to $-\infty$. This shows that the expression converges to 0.

(c)

$$\lim_{x\to-\infty}\frac{3x^2-6x+2}{2x^2-9} =$$

Solution: This one is like the last one:

$$\begin{aligned}\lim_{x\to-\infty}\frac{3x^2-6x+2}{2x^2-9} &= \lim_{x\to-\infty}\frac{3x^2-6x+2}{2x^2-9}\left(\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right)\\ &= \lim_{x\to-\infty}\frac{3-\frac{6}{x}+\frac{2}{x^2}}{2-\frac{9}{x^2}} = \frac{3}{2}.\end{aligned}$$

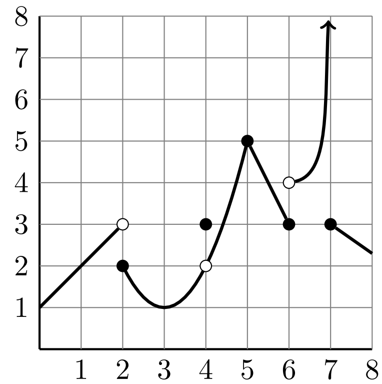
(d)

$$\lim_{t\to 3}\frac{t^2-t+1}{2t^2-8} =$$

Solution: The function is continuous at $t = 3$ and we can substitute to find the answer:

$$\lim_{t\to 3}\frac{t^2-t+1}{2t^2-8} = \frac{3^2-3+1}{2(3)^2-8} = \frac{7}{10}.$$

[18 points] 2. Here is the graph of a function $f(x)$ on the domain $0 \le x \le 8$:



In the following questions, you do not need to justify your answers.

(a) List all values of x where $f'(x) = 0$.

Solution: This is all places where the tangent line to the curve is horizontal. The only one is $x = 3$.

(b) Compute the following limits. If they don't exist, say so.

(i) $\lim_{x\to 2} f(x) =$ doesn't exist

(ii) $\lim_{x\to 4} f(x) = 2$

(iii) $\lim_{x\to 5} f(x) = 5$

(iv) $\lim_{x\to 6^-} f(x) = 3$

(v) $\lim_{x\to 6^+} f(x) = 4$

(vi) $\lim_{x\to 7} f(x) =$ doesn't exist

(c) Circle true or false in the following questions. You do not need to justify your answers.

(i) $f(x)$ is continuous at $x = 1$.	**True**	False
(ii) $f(x)$ is continuous at $x = 2$.	True	**False**
(iii) $f(x)$ is continuous at $x = 3$.	**True**	False
(iv) $f(x)$ is continuous at $x = 4$.	True	**False**
(v) $f(x)$ is continuous at $x = 5$.	**True**	False
(vi) $f(x)$ is continuous at $x = 6$.	True	**False**
(vii) $f(x)$ is continuous at $x = 7$.	True	**False**

(d) Estimate the following derivatives as best you can from the graph. If the derivative does not exist, say so.

(i) $f'(1) = 1$

(ii) $f'(2) =$ doesn't exist

(iii) $f'(3) = 0$

[16 points] 3. (a) Find the equation for the tangent line to the graph $y = x^2 - 5x + 1$ at $x = 3$.

Solution: The derivative of this function is

$$y' = 2x - 5.$$

At $x = 3$, this is equal to 1. So, the tangent line at $x = 3$ has slope 1. It contains the point $(3, -5)$. Using the point-slope form, it's

$$y + 5 = x - 3,$$

which you could also write as $y = x - 8$.

(b) Compute $\dfrac{d}{dt}\left(\dfrac{e^t}{t^3}\right)$.

Solution: Using the quotient rule, it's

$$\frac{d}{dt}\left(\frac{e^t}{t^3}\right) = \frac{t^3 e^t - e^t(3t^2)}{(t^3)^2} = \frac{(t^3 - 3t^2)e^t}{t^6} = \frac{(t-3)e^t}{t^4}.$$

It's not important to me whether you simplified it all after applying the quotient rule.

[10 points] 4. Let $f(t) = \frac{4}{1+5t}$. Directly using the definition of the derivative, find $f'(-1)$.
Warning: You will receive no credit for using differentiation rules. You must use limits to compute the derivative directly from the definition.

Solution: The difference quotient is

$$\begin{aligned}\frac{f(-1+h) - f(-1)}{h} &= \frac{\frac{4}{1+5(-1+h)} - \frac{4}{1+5(-1)}}{h} \\ &= \frac{\frac{4}{-4+5h} - (-1)}{h} \\ &= \frac{4 + (-4+5h)}{h(-4+5h)} \\ &= \frac{5h}{h(-4+5h)} = \frac{5}{-4+5h}.\end{aligned}$$

So,

$$f'(-1) = \lim_{h\to 0} \frac{f(-1+h) - f(-1)}{h} = \lim_{h\to 0} \frac{5}{-4+5h} = -\frac{5}{4}.$$

[14 points] 5. Let $g(x) = x^3 + 3x^2 - 9x + 1$.

(a) What is the average rate of change of $g(x)$ from $x = 0$ to $x = 2$?

Solution: Plug in to find that $g(0) = 1$ and $g(2) = 3$. The average rate of change is then

$$\frac{g(2) - g(0)}{2 - 0} = \frac{3 - 1}{2 - 0} = 1.$$

(b) Find all x-coordinates where the tangent line to $g(x)$ is horizontal.

Solution: We need to find all x such that $g'(x) = 0$. First, we find the derivative:

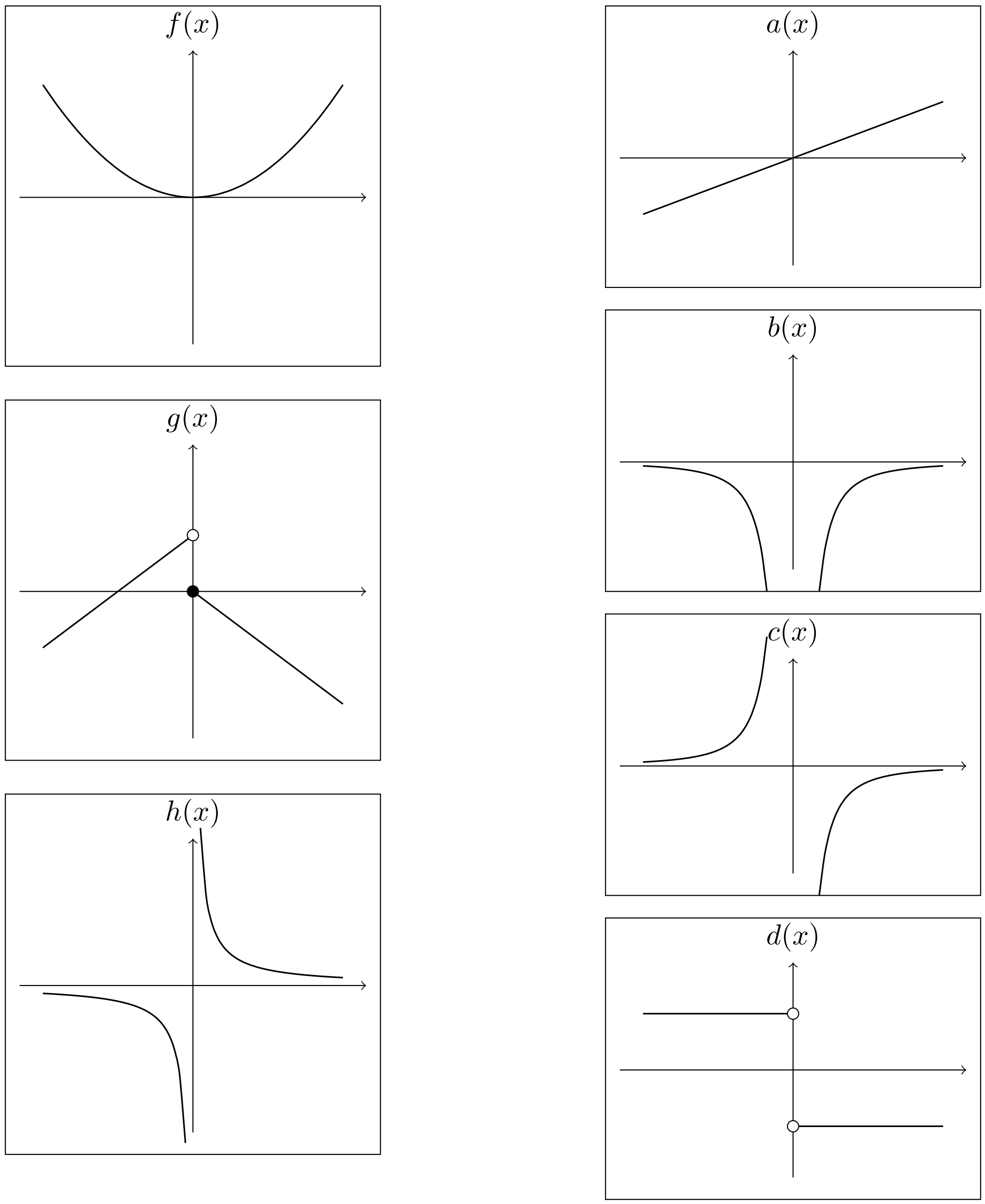
$$g'(x) = 3x^2 + 6x - 9.$$

We factor and set this equal to 0 and solve:

$$3x^2 + 6x - 9 = 3(x + 3)(x - 1) = 0.$$

So, the tangent line is horizontal when $x = -3$ or $x = 1$.

[9 points] 6. On the left are three functions $f(x)$, $g(x)$, and $h(x)$. On the right are four functions, $a(x)$, $b(x)$, $c(x)$, and $d(x)$.



Circle one answer for each question:

(a) The derivative of $f(x)$ is… $\mathbf{a}(\mathbf{x})$ $b(x)$ $c(x)$ $d(x)$ none of the above

(b) The derivative of $g(x)$ is… $a(x)$ $b(x)$ $c(x)$ $\mathbf{d}(\mathbf{x})$ none of the above

(c) The derivative of $h(x)$ is… $a(x)$ $\mathbf{b}(\mathbf{x})$ $c(x)$ $d(x)$ none of the above

This page can be used as scratch paper.