1. If x is a number, you multiply it by itself, subtract five from this, and then divide the result by 2, what do you get? (Your answer will be in terms of x, of course.)

Solution: Translating the words into algebra, it's
$$\frac{x^2-5}{2}.$$

2. How can you expand/simplify the following expressions? Don't just give the answer from memory—explain (to yourself) why they're true. Note that for some of them, there's no good way to expand or simplify the expression.

   (a) $(xy)^2$

   Solution: It's x^2y^2. It's because $(xy)^2 = (xy)(xy) = xyxy = x^2y^2$, since you can multiply numbers in any order.

   (b) $a(b+c)$

   Solution: It's equal to $ab + ac$. This is called the distributive property of multiplication. To see why it's true, let's imagine that b and c represent the number of blue and cyan objects we have, just to make things a bit more concrete. Now imagine we have a total of a copies of these $b+c$ many objects, for a total of $a(b+c)$. Then we have ab blue objects and ac cyan objects, for a total of $ab+ac$. So $a(b+c) = ab+ac$.

   (c) $(a+b)(c+d)$

   Solution: This is equal to $ac+ad+bc+bd$. This is something we typically memorize because we use it so often, but it comes from applying the distributive property above twice:
   $$(a+b)(c+d) = (a+b)c + (a+b)d = ac+bc+ad+bd$$

   (d) $(x+y)^2$

   Solution: It's equal to $x^2+2xy+y^2$. This comes from the previous equation:
   $$(x+y)(x+y) = xx+xy+yx+y^2 = x^2+2xy+y^2.$$

   (e) $\sqrt{x+y}$

Solution: There is no way to simplify this. In particular, it is **not equal** to $\sqrt{x}+\sqrt{y}$. Just try it with $x=9$ and $y=16$. Then $\sqrt{x}=3$ and $\sqrt{y}=4$, but $\sqrt{x+y}=\sqrt{25}=5$, which is definitely not tthe same as $3+4$.

3. Does the equation $x^2+y^2=1$ represent any geometric shape? In what way?

Solution: It represents a circle with radius 1 in the two-dimensional plane around the origin, in the sense that the set of points (x,y) satisfying the equation forms a circle. This comes from the Pythagorean theorem: the distance from a point (x,y) to the origin $(0,0)$ is $\sqrt{(x-0)^2+(y-0)^2}=\sqrt{x^2+y^2}$, and so the points satisfying $x^2+y^2=1$ are exactly those at distance 1 from the origin.

4. Solve the following equation:

$$\frac{5}{2d-4}=9$$

Solution: Note that the fraction is undefined when $2d-4=0$. Assuming this isn't the case, multiply both sides by $2d-4$ to obtain an equivalent equation

$$5=9(2d-4)=18d-36.$$

(What does "equivalent equation" mean? See the next problem.) Now add 36 to both sides of the equation to obtain

$$41=18d.$$

Now divide both sides of the equation by 18 to get

$$d=\frac{41}{18}.$$

5. Here are some pairs of equations. For each pair, are the two equations equivalent to each other? For that matter, what does it mean for two equations to be equivalent to each other?

   (a) $x+y+z=1$ and $x+y=1-z$

Solution: Two equations are equivalent if whenever one of them is true, the other is true. And yes, these two equations are equivalent to each other. We can derive one equation from the other by adding or subtracting z to each side of the equation. (This is how we get equivalent equations, typically: by doing the same thing to each side of an equation.)

(b) $x^2 + x = 0$ and $x + 1 = 0$

Solution: These are not quite equivalent. The first equation has solutions $x = 0$ and $x = -1$, while the second only has solution $x = -1$.

The two equations are related in that you can get the second from the first by dividing both sides by x, so long as $x \neq 0$. This means that if the first equation holds and $x \neq 0$, then the second equation holds (which is true!). But when the first equation holds and $x = 0$, the second equation does not hold.

Moral: be careful when dividing both sides of an equation by a variable. Remember that in doing this step, you're assuming that the variable is nonzero.

(c) $2x^2 + 4x + 6 = 0$ and $x^2 + 2x + 3 = 0$

Solution: These are equivalent. The second is obtained from the first by dividing both sides of the equation by 2, and the first is obtained from the second by multiplying both sides of the equation by 2. Thus they have the same solutions.